<table>
<thead>
<tr>
<th>Title</th>
<th>A MODEL OF THE CYCLICAL GROWTH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Author(s)</td>
<td>Okamoto, Takeyuki</td>
</tr>
<tr>
<td>Editor(s)</td>
<td></td>
</tr>
<tr>
<td>Citation</td>
<td>Bulletin of University of Osaka Prefecture. Ser. D, Sciences of economy, commerce and law. 1963, 7, p.75-79</td>
</tr>
<tr>
<td>Issue Date</td>
<td>1963-03-20</td>
</tr>
<tr>
<td>URL</td>
<td><a href="http://repository.osakafu-u.ac.jp/dspace/">http://repository.osakafu-u.ac.jp/dspace/</a></td>
</tr>
<tr>
<td>Rights</td>
<td></td>
</tr>
</tbody>
</table>
A MODEL OF THE CYCLICAL GROWTH

Takeyuki Okamoto

1. In this paper an attempt is made to present a cyclical growth model of Kaldor type. Mr. Kaldor in his essay entitled "A Model of the Trade Cycle" treated a pure trade cycle. There is little doubt, however, that the trade cycle we know is conditioned by the occurrence in a growing economy; therefore the basic ideas of his essay ought to be applied to both trade cycle and economic growth problems.

2. We will work with a slightly modified version of Kaldor's model, and assume that

\[ (1) \quad I = I(Y, K) + A; \quad \frac{\partial I}{\partial Y} > 0, \quad \frac{\partial I}{\partial K} < 0 \]

\[ (2) \quad S = S(Y, K) - B; \quad \frac{\partial S}{\partial Y} > 0, \quad \frac{\partial S}{\partial K} > 0 \]

where \( Y \) stands for gross national output or real income; \( I \), gross real investment; \( S \), gross real savings; \( K \), real capital stock; \( A \), autonomous investment; \( B \), basic consumption.

In Kaldor's model it is assumed that the functional forms of \( I \) and \( S \) are both non-linear. But, we assume, for simplicity, the \( S \) function to be linear, though the \( I \) function to be non-linear in like manner with Kaldor's.

3. Given the amount of real capital, there is a "normal" value of \( \frac{\partial I}{\partial Y} \) appropriate for "normal" levels of \( Y \). If incomes are much below them, \( \frac{\partial I}{\partial Y} \) will be small relatively to its "normal" value or zero, because there is a great deal of surplus capacity. If incomes are much above them, \( \frac{\partial I}{\partial Y} \) will also be small or zero, owing to the rising costs of construction, increasing costs and increasing difficulty of borrowing. Thus the functional form of \( I \) is to be non-linear as shown in Fig. 1.


4. The accumulation of capital, by restricting investment opportunities, will tend to make investment fall. On the other hand, however, the accumulation of capital, by increasing capacity, will tend to make the "normal" level of $Y$ rise and to make its range larger. Therefore, when the level of investment is high, including given autonomous investment, the $I$ curve gradually shifts to the right, and its ceiling is gradually placed above as shown in Fig. 1; on the contrary, when the level of investment is low — strictly speaking, net investment is negative though gross investment is not zero, the direction of shift of the $I$ curve should be in the opposite.

We assume that the autonomous investment does not alter the functional form of $I$, so that the autonomous investment determines only the position of the $I$ curve.

5. As to the $S$ function we assume $\frac{\delta S}{\delta Y}$ to be constant, and $\frac{\delta S}{\delta K}$ to be negligibly small. This is assumed merely for simplicity; therefore we are not in any sense positive in denying the fact that $\frac{\delta S}{\delta Y}$ will be large, both for low and for high levels of $Y$, as compared with its "normal" value, and $\frac{\delta S}{\delta K}$ will be positive. Thus the $S$ function is assumed to be a straight line as shown in Fig. 2.

Now, the magnitude of basic consumption depends on the size of population and the standard of living. Even if there is no population growth, the standard of living may rise as civilization and economic condition progress, and it will not be cut down its level once arrived except for emergency. The basic consumption, of course, gradually increases when population grows steadily. Thus, as the basic consumption which is assumed to be an increasing function of time determines the position of the $S$ curve, the increase in the basic consumption causes the $S$ curve to shift downwards.

6. Next, as in the Kaldor's model, we also assume further that the "normal"

---

3 N. Kaldor, _ibid._, pp. 180–182.
value of $\frac{\Delta I}{\partial Y}$ is greater than the value of $\frac{\Delta S}{\partial Y}$. On this and the above assumptions, investment and savings in (1) and (2) appear as shown in Fig. 3. Needless to say, the meeting points of the $I$ curve and the $S$ line are positions of short-period equilibrium; both $a$ and $b$ (in Fig. 3) are stable, but $c$ is unstable.

7. Well, let us begin by explaining the pure trade cycle of Kaldor type. The dotted line $RR$ shown in Fig. 4 represents "the locus of points on the $I$ curves where the level of investment decisions corresponds to replacement so that net investment is zero".\(^4\) If the autonomous investment $A$ is given and is smaller than replacement, and if the $S$ line, say $S_1$, does not shift anywhere for a while, the cyclical movement of the system will be indicated by the trajectory $abcde$, for "economic activity always tends towards a level where Savings and Investment are equal"\(^5\) and $\frac{\Delta I}{\partial Y}$ is smaller than $\frac{\Delta S}{\partial Y}$. Even if we start from any position on the $I$ curves outside or inside the trajectory, the system will move on to it. We can say this is a model of the pure trade cycle of Kaldor type.

8. Now, let $R'R'$ instead of $RR$ (in Fig. 4) represent the levels of replacement. Hence the given autonomous investment $A$ is larger than replacement. In this case the cyclical movement of the system does not arise. If we start from any point, the system will stay at the position $a$, for the $I$ curve will continuously shift to the right. We can say this is the situation given the name of "poverty in the midst of plenty"\(^6\) — underemployment equilibrium in the sense of Keynes.

\(^4\) N. Kaldor, *ibid.*, p. 819.
\(^5\) N. Kaldor, *ibid.*, p. 177.
9. Thus far, we have abstracted out the shift of the $S$ curve as if the basic consumption were constant over time. Let us now introduce the shift of the $S$ curve into our system in order to see what effects the increase in basic consumption would have on the growth of income.

![Diagram](image)

Fig. 5

Since the basic consumption is assumed to be an increasing function of time, the $S$ curve, say $S_1$ in Fig. 5, can not remain the same in course of time; the $S$ line will shift from $S_1$ to $S_2$, and to $S_3$ while the $I$ curve shifts from $I_1$ to $I_2$ and to $I_3$. Therefore, new positions of short-period equilibrium, say $f$ and $g$ in Fig. 5, can be reached before such a cyclical movement as indicated by the trajectory $abcde$ occurs. Here we can find the very process of income expansion which is derived by the increase in basic consumption.

10. We are now ready to analyse the cyclical growth of our system. It is clear that the movement of cyclical growth of the system depends on the rate at which the $I$ and $S$ curves shift at any particular level of investment. If the rate of shift of the $S$ curve is greater than that of the $I$ curve, the growth of income of the economy will be continued steadily; on the contrary, if the rate of shift of the $S$ curve is smaller than that of the $I$ curve, it will not be continued and the cyclical movement of the system will occur.

The rate at which the $I$ curve shifts seems to depend on the construction period and durability of capital goods. The shorter the construction period and the longer the life-time of capital goods, the larger will be the stock of capital at a given rate of investment. Hence the shorter the construction period and the higher the durability, the faster will be the rate of (right-upward) shift of the $I$ curve.\(^7\)

---


Mr. Kaldor holds a different view: "the shorter the construction period, and the lower the durability, the faster will be the rate of shift of the $S$ and $I$ curves at any given rate of investment". However, the shorter the life-time of capital goods, the smaller will be the stock of capital and the more entrepreneurs will tend to invest; so that the $I$ curve will shift left-upwards.
We shall now drop the assumption that the autonomous investment is constant and introduce the autonomous investment as an increasing function of time into our system, for it will depend on technical progress which will be accelerated in process of time. Hence the flooring of the $I$ curve gradually rises.

Now, let us assume that the rate of shift of the $I$ curve is faster than that of the $S$ curve. And to clinch our understanding of the cyclical growth, let us again make diagramatic analysis. If we start from the point $a$ on the $I_1$ curve shown in Fig. 6, the cumulative forces will increase income and investment decisions until the system reaches $b$, and thereafter activity will move upwards (owing to the gradual right-upward shift of the $I$ curve) along the $S_1$ line until it reaches $c$. At that point short-period equilibrium becomes unstable, and a downward moving cumulative process is set up. On the way downward the shift of the $S$ line from $S_1$ to $S_2$ will occur owing to a increase in basic consumption which is produced by the experience of high consumption in the preceding boom process $b$-$c$. Hence the downward moving cumulative process lands the system at $d$, where investment is less than replacement. Thereafter activity will move upwards (owing to the gradual left-upward shift of the $I$ curve which is caused by the gradual reduction in available equipment and the gradual increase in autonomous investment) along the $S_2$ line until it reaches $e$. At that point short-period equilibrium again becomes unstable, and an upward cumulative movement along the $I_3$ curve follows which lands the system at $f$.

Here we can find a model of the cyclical growth. There appears, of course, to be no necessary reason why the cyclical growth should occur; either a cyclical growth or a steady growth should happen to occur depending on the rate at which the $I$ and $S$ curves shift.