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Minoru Tachibana

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School of Economics

Osaka Prefecture University

Sakai, Osaka 599-8531, Japan

Safe haven government bonds: Identification using a regime-switching copula model*

Minoru Tachibana

Graduate School of Economics, Osaka Prefecture University

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Abstract

The purpose of this paper is to identify safe haven government bonds against domestic stock markets and against internationally diversified stock portfolios. We develop a two-state regime-switching copula model that describes various patterns of the stock–bond return dependence structure and apply it to eight developed countries (Australia, Canada, France, Germany, Japan, Switzerland, the United Kingdom, and the United States) for the period 1999 to 2018. We find that government bonds of all of the countries (except for Canada) qualify as safe havens for domestic stock markets. However, for international stock portfolios, only US government bonds were a safe haven for the entire period while German, Japanese, and UK government bonds served as safe havens for certain limited periods. The decomposition analysis shows that the correlation between local currencies and international stock markets is an important factor in determining safe haven government bonds against international stock portfolios.

Keywords: safe haven government bond, stock market, regime-switching copula model.

JEL Classification: C58, G11, G15.

*I acknowledge financial support from JSPS KAKENHI (grant number: 16K03746). E-mail address: mtachi@eco.osakafu-u.ac.jp

1 Introduction

Understanding the relationship between stock and government bond markets is crucial for investors because trading volumes of the two asset classes are large and their risk-return profiles are quite distinct. Particularly, under conditions of economic or financial crises, stock and government bond prices tend to move in opposite directions due to flight-to-quality behavior from stocks to bonds. This negative co-movement between stocks and government bonds, if any, can reduce the risk of substantial loss in portfolio values during periods of crisis. Thus, identifying the relationship of the two asset classes, particularly during periods of financial market turbulence, would have practical implications for asset allocation and risk management.

This paper investigates the dynamic relationship between stock and government bond returns and is primarily interested in the relationship under extraordinary financial circumstances. Specifically, our main purpose is to identify *safe haven government bonds*, which are defined as government bonds with returns that are negatively correlated with stock returns in times of market stress or turmoil. We develop a two-state regime-switching copula model to specify the stock–bond return dependence structure and present statistical methods to determine safe haven government bonds. We then apply the methodology to stock and government bond return data of eight developed countries (Australia, Canada, France, Germany, Japan, Switzerland, the United Kingdom, and the United States) for the period 1999 to 2018.

The traditional present value models of stock and bond price formations predict both positive and negative relations between the two variables. First, variation in real interest rates generates a positive correlation since both stock and bond prices are affected by a common part of the discount rates. Second, a real shock to expected future dividends might cause a negative correlation since future cash flows from bonds are fixed regardless of the shock. Third, variation in expected inflation could induce both a positive and negative correlation. This is because an inflationary shock has a negative impact on bond prices through higher nominal discount rates while the effect is ambiguous for stock prices due to simultaneous changes in nominal discount rates and expected nominal dividends. Fourth, financial crisis episodes could also create both a positive and negative relation. On the one hand, an extreme negative shock to stock markets would promote a negative correlation as the required risk premiums for stocks increases and the risk premiums for government bonds decline, which corresponds to the flight-to-quality phenomenon. On the other hand, when the extreme shock comes from fear of sovereign default such as the recent Greek debt crisis, there would be a positive correlation due to increases in risk premiums for both stocks and bonds, which represents contagion between stock and bond markets.

A vast number of empirical studies have investigated the relationship between stock and bond markets and its economic sources. Among the studies, several authors document evidence on the existence of flight-to-quality from stocks to bonds for major developed countries. For instance, Gulko (2002) finds that the correlation between the returns of US stocks and Treasury bonds switches its sign from positive to negative during stock market crashes. Connolly et al. (2005) show that US Treasury bond returns tend to be high relative to US stock returns during days when the implied volatility from equity index options increases substantially and during days when stock turnover is unexpectedly high. Connolly et al. (2007) present similar results for Germany and the United Kingdom. Analyzing stock–bond return linkages for eight developed countries (Australia, Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States), Baur and Lucey (2009) show that flights (flight-to-quality from stocks to bonds and flight-from-quality from bonds to stocks) exist and occur at the same time in many countries. Other studies that find evidence on flight-to-quality include Andersson et al. (2008), Aslanidis and Christiansen (2012), Bansal et al. (2010), Baur and McDermott (2013), Chiang et al. (2015), Ciner et al. (2013), Dajcman (2012), Hartmann et al. (2004), Ilmanen (2003), Kim et al. (2006), and Ohmi and Okimoto (2016).[1]

[1] In contrast to evidence on flight-to-quality, diverse results are reported in the literature with regard to the effects of macroeconomic fundamentals (e.g., changes in expectations of inflation, output growth, and interest rates) on stock–bond co-movements. For this point, see and compare results in Andersson et al. (2008), Aslanidis and Christiansen (2012), Baele et al. (2010), Christiansen and Ranaldo (2007), Hong et al. (2014), Ilmanen (2003), Li (2002), and Yang et al.(2009). With regard to contagion between stock and bond markets, there are only a few studies, and they show different consequences: Hartmann et al. (2004) show that stock–

In recent years, copula-based approaches have been adopted in literature to study the relationship between stock and bond markets (Chang and Hsueh, 2013; Chen et al., 2014; Chui and Yang, 2012; Durand et al., 2010; Garcia and Tsafack, 2011; Jammazi et al., 2015; Nguyen and Liu, 2017; Wu and Lin, 2014). The advantage of using a copula approach is that it allows for flexibility when specifying a model for the joint distribution of stock and bond returns. In addition, the tail dependence coefficient of a copula is a useful measure to determine safe haven bonds as it represents the probability of observing extreme values in variables of interest.

This paper relies on a copula-based approach to identify safe haven government bonds and contributes to the literature in three ways. First, we develop a two-state regime-switching copula model that describes various patterns of the stock–bond return dependence structure including a combination of positive and negative dependence regimes that theories predict. Following Patton (2006), most of the copula studies presented in the previous paragraph allow for time variation in the stock–bond dependence by using ARMA-type specifications. However, this type of the time-varying specification is restrictive because it assumes that although the parameters associated with a copula function vary over time, the functional form of the copula remains fixed over a sample period. Copulas typically describe only one direction of the dependence between variables (usually positive dependence and, when rotated by 90 or 270 degrees, copulas describe negative dependence) except for some copulas such as Gaussian or Student's t copula. Therefore, fixing a copula function over a sample period means that researchers confine themselves to analyzing only positive (or negative) dependence. This does not matter when examining the dependence structure between assets within the same class (e.g., dependence structure between stocks),[2] but it is a serious problem

bond contagion is approximately as frequent as flight-to-quality from stocks to bonds in G-5 countries, whereas Baur and Lucey (2009) find that stock–bond contagion only existed during the crisis period of the 9/11 attacks in 2001 out of six crisis periods for three countries out of eight.

[2]Early studies that introduce a regime-switching framework into copula are Chollete et al. (2009), Okimoto (2008), and Rodriguez (2007). These authors' models are restrictive in terms of a description of negative dependence compared to a description of positive dependence. However, because these studies investigate dependence structures in international stock markets, there is less need to consider flexible and comprehensive modeling for negative dependence.

when analyzing the relationship between variables that have potentially both positive and negative dependence over a sample period as in the case of the stock–bond relations. In addition to the positive–negative dependence regimes, our regime-switching copula model can describe positive–positive and negative–negative dependence regimes. Our model also captures various possible characteristics of the joint distribution of stock and bond returns in terms of asymmetry and tail dependence by using five different copulas for each regime (Gaussian, Student's t, Clayton, Gumbel, and BB7 for a positive dependence regime and their 90-degree clockwise rotations for a negative dependence regime).

Among the above-mentioned studies that examined stock–bond co-movements using copulas, Garcia and Tsafack (2011) is the only article that employed a regime-switching copula model. Their model consists of two regimes, normal and asymmetric regimes, the former specified by a Gaussian copula and the latter by the mixture of survival (180-degree rotated) Gumbel copulas. This means that negative dependence, if any, is captured only by the Gaussian copula in their regime-switching copula model, which is restrictive because the Gaussian copula is symmetric with no tail dependence. The closest model to ours is the two-state regime-switching copula model that Wang et al. (2013) developed to investigate the dependence structure between stock and foreign exchange markets. The two states in their model correspond to positive and negative dependence regimes, and each of the regimes is specified by the mixture of the Clayton and survival Clayton copulas. On the other hand, our regime-switching copula model can describe more various dependence patterns by considering positive–positive and negative–negative dependence regimes as well as positive–negative regimes and by specifying each regime with each of the five copulas that have different types of asymmetry and tail dependence.

The second contribution of this paper is that after estimating the regime-switching copula model, we use two distinct statistical methods to identify safe haven government bonds. The first method relies on the lower–upper tail dependence, which measures the probability of observing both extreme negative stock returns and extreme positive government bond returns.

The second is based on a regression of the smoothed Kendall's τ rank correlation between stock and government bond returns on the measure of stock market volatility. As explained in detail in Section 2, both methods are consistent with the definition of a safe haven government bond, although they emphasize different aspects of safe haven properties. The availability of both of the two tests are specific to the regime-switching copula models: the ARMA-type time-varying specifications proposed by Patton (2006) and used often in the existing copula work cannot conduct statistical inference for the tail dependence, whereas static copula models in which dependence structure between variables is assumed to be constant cannot use the latter regression-based test. For this reason, only one of the two methods or no statistical method has been employed in the previous copula studies to determine safe haven bonds. In this paper, these two statistical methods are also used to test the presence of *contagion* between stock and government bond markets, and a variant of the second method is used to examine whether government bonds under study served as *hedges* against stock markets.

The third contribution is that we identify safe haven government bonds against domestic stock markets and against internationally diversified stock portfolios. To our best knowledge, the latter type of safe haven government bonds is only examined in Baur and McDermott (2013) among existing work.[3] Given the fact that diversification opportunities across international stock markets have been expanding in recent years, our analysis using the regime-switching copula model would provide additional useful information for investors. Moreover, after determining safe haven government bonds against international stock portfolios, we conduct decomposition analysis in which government bond returns of each country that are denominated in the US dollar are divided into government bond returns expressed in the country's currency and the currency returns. By doing so, we find which of the two factors, appreciation in government bond price itself or appreciation in currency value at times of financial crisis, has a greater influence on the determination of the international safe haven roles.

[3] Using VAR models, Baur and McDermott (2013) investigate whether US government bonds and gold are safe haven assets against the global stock market.

The remainder of the paper is structured as follows. Section 2 introduces our regime-switching copula model and statistical methods to identify safe haven government bonds. Section 3 presents the data and the empirical results. Section 4 concludes.

2 Empirical methodology

In this section, we first develop a two-state regime-switching copula model. We then specify marginal models for stock and government bond returns and explain our estimation procedure for the regime-switching copula model. Finally, we present two different statistical methods to identify safe haven government bonds.

2.1 The regime-switching copula model

Let X_1 and X_2 be random variables of stock and government bond returns, respectively. F denotes the joint distribution function of X_1 and X_2 with marginal distribution functions F_1 and F_2. Then, according to Sklar's theorem, there exists a copula function C such that

$$F(x_1, x_2) = C(F_1(x_1), F_2(x_2)). \tag{1}$$

The copula function is also expressed as

$$C(u_1, u_2) = F\left(F_1^{-1}(u_1), F_2^{-1}(u_2)\right), \tag{2}$$

where $F_i^{-1}(u_i)$ for $i = 1, 2$ are the inverse marginal distributions. Hence, the copula function is regarded as a joint distribution function of U_1 and U_2; that is, $C(u_1, u_2) = \Pr(U_1 \leq u_1, U_2 \leq u_2)$ where U_i are defined as $U_i = F_i(X_i)$ for $i = 1, 2$ (probability integral transformations) and thus follow a standard uniform distribution, $U_i \sim U(0, 1)$. Eqs.(1) and (2) suggest that the dependence structure between variables is described by a copula and separated from marginal distributions of the variables.[4]

As discussed in the Introduction, theoretical models predict both positive and negative relations between stock and government bond returns. Therefore, when modeling the dependence structure between the two variables, we need to consider possible shifts between positive and negative dependence. However, copula functions developed in the literature only describe either positive or negative dependence between variables except for a few copulas such as Gaussian or Student's t copula. For this reason, we introduce a two-state regime-switching copula model with one state being a positive dependence regime and the other a negative dependence regime (positive–negative dependence regimes). Of course, such a specification is not appropriate if the sign of the dependence has not changed throughout the sample period. Our regime-switching copula model allows for such a case with positive–positive or negative–negative dependence regimes.

Specifically, let $C(u_{1t}, u_{2t}|s_t)$ be a state-dependent copula function at time t defined as

$$C(u_{1t}, u_{2t}|s_t) = \begin{cases} C_1(u_{1t}, u_{2t}), & \text{if } \; s_t = 1, \\ C_2(u_{1t}, u_{2t}), & \text{if } \; s_t = 2, \end{cases} \tag{3}$$

where s_t is an unobserved state variable and $C_1(u_{1t}, u_{2t})$ and $C_2(u_{1t}, u_{2t})$ are copula functions under state 1 and state 2, respectively. In addition, the state variable is assumed to follow a Markov chain with the transition probability matrix given by

$$P = \begin{bmatrix} p_{11} & 1 - p_{22} \\ 1 - p_{11} & p_{22} \end{bmatrix}, \tag{4}$$

where transition probability p_{ij} is defined by $p_{ij} = \Pr(s_t = j|s_{t-1} = i)$ for $i, j = 1, 2$.

We assume that a pair of state 1 and state 2 represents one of the three different combinations: positive–negative, positive–positive, or negative–negative dependence regimes. For each positive dependence regime, we consider five different copulas (Gaussian with a positive

[4] See Joe (1997, 2015) and Nelsen (2006) for a comprehensive exposition of copulas.

linear correlation, Student's t with a positive linear correlation, Clayton, Gumbel, and BB7). For each negative dependence regime, we use 90-degree clockwise rotations of the five copulas, which are defined as $C_{90}(u_1, u_2) = u_1 - C(u_1, 1 - u_2)$.

[Table 1 around here]

The use of these 10 copulas enables us to describe possible various patterns of the dependence structure between stock and government bond returns. Table 1 presents functional forms of the five copulas (used for a positive dependence regime) followed by corresponding functions of Kendall's τ and the lower and upper tail dependence, which are all defined as a function of the copula parameter. Here, Kendall's τ measures a rank correlation between two variables of interest while the lower (upper) tail dependence indicates the probability that two variables jointly take extreme negative (positive) values. As the last two columns of Table 1 show, the five copulas have different structures in terms of tail dependence: Gaussian copula has no tail dependence; Student's t copula has the same degree of lower and upper tail dependence; Clayton copula has only lower tail dependence; Gumbel copula has only upper tail dependence; and BB7 copula has both upper and lower tail dependences that are allowed to be different. Moreover, Gaussian and Student's t copulas are symmetric distributions, whereas Clayton, Gumbel, and BB7 copulas are asymmetric distributions. The same properties hold when these copulas are rotated by 90 degrees clockwise to describe negative dependence. Therefore, our two-state regime-switching copula model takes into account many possible dependence structures between stock and government bond returns in terms of positive or negative dependence, symmetric or asymmetric dependence, and tail or no tail dependence. There is a total of 55 combinations of copulas for C_1 and C_2,[5] and we select the best combination among them based on the Akaike information criterion (AIC).

[5] There are 25 combinations for positive–negative dependence regimes and 15 combinations for each of positive–positive and negative–negative dependence regimes.

2.2 Specification of marginal models

We next specify marginal models of stock returns (X_1) and government bond returns (X_2). In this study, they are characterized by a class of ARMA(p,q)–GJR–GARCH(1,1) models as follows:

$$
\begin{aligned}
X_{it} &= \alpha_i + \sum_{l=1}^{p} \beta_{il} X_{i,t-l} + \sum_{l=1}^{q} \gamma_{il} \varepsilon_{i,t-l} + \varepsilon_{it}, && (5)\\
\varepsilon_{it} &= \sigma_{it} z_{it}, && (6)\\
\sigma_{it}^2 &= \phi_{i0} + \phi_{i1}\sigma_{i,t-1}^2 + \phi_{i2}\varepsilon_{i,t-1}^2 + \tilde{\phi}_{i2} D_{i,t-1}\varepsilon_{i,t-1}^2, && (7)
\end{aligned}
$$

for $i = 1, 2$ and $t = 1, ..., T$. The ARMA model (5) and the GARCH model (7) represent conditional mean and variance of the marginal distribution of X_{it}, respectively. Following Glosten et al. (1993), the GARCH model allows for the leverage effect by adding the so-called GJR term to the right-hand side of Eq.(7) where the dummy variable $D_{i,t-1}$ takes the value of 1 if $\varepsilon_{i,t-1}$ is negative and 0 otherwise.

We assume that the standardized disturbance z_{it} in Eq.(6) is an independently and identically distributed (i.i.d.) random variable that follows the standard normal, Student's t, or skewed t distribution. The Student's t distribution can model well-known fat tails of financial return data with the degree of freedom parameter. The skewed t distribution proposed by Hansen (1994) describes the asymmetry of financial return data with the skewness parameter $\zeta \in (-1, 1)$ as well as fat tails with the degree of freedom parameter ν. It reduces to the Student's t distribution when $\zeta = 0$, to the skewed normal distribution when $\nu \to \infty$, and to the normal distribution when $\zeta = 0$ and $\nu \to \infty$.

As explained in more detail in Section 3.2, lag orders of the ARMA model, p and q, are set to either 0 or 1, which leads to a total of 24 marginal models for each return series. Among them, the best marginal model is selected based on AIC as in the selection of the best combination of copula functions.

2.3 Estimation procedure

Estimation of the regime-switching copula model is implemented through the maximum likelihood (ML) method. The log-likelihood function at time t is easily derived by modifying Eq.(1) for the analysis of time-series data,[6] differentiating both sides of the modified equation with respect to x_{1t} and x_{2t} and taking the logarithm. The resulting log-likelihood at time t is given by

$$\begin{aligned}\log f(x_{1t}, x_{2t}|\Omega_{t-1};\theta) &= \log f_1(x_{1t}|\Omega_{t-1};\theta_1) + \log f_2(x_{2t}|\Omega_{t-1};\theta_2) \\ &\quad + \log c(u_{1t}, u_{2t}|\Omega_{t-1};\theta_c), \end{aligned} \tag{8}$$

where Ω_{t-1} denotes the information set available at the end of time $t-1$, f the conditional joint density of X_{1t} and X_{2t}, f_i the conditional marginal density of X_{it} for $i = 1, 2$, c the conditional copula density, and $u_{it} = F_i(x_{it}|\Omega_{t-1};\theta_i)$ for $i = 1, 2$. θ_1 and θ_2 are parameter vectors for conditional marginal models of X_{1t} and X_{2t}, respectively, θ_c is the parameter vector for a copula, and all of the parameters are collected as $\theta = [\theta_1', \theta_2', \theta_c']'$.

The relatively large dimension of the parameters makes it difficult to achieve maximization of the log-likelihood function. Therefore, we employ the two-stage ML estimation or the "inference functions for margins" (IFM) method proposed by Joe and Xu (1996). In the first stage, the parameters of each of the conditional marginal distributions are estimated separately by ML:

$$\hat{\theta}_i = \arg\max_{\theta_i} \sum_{t=1}^{T} \log f_i(x_{it}|\Omega_{t-1};\theta_i), \tag{9}$$

for $i = 1, 2$. Then, in the second stage, copula parameters are estimated by ML given $\hat{\theta}_1$ and

[6]Patton (2006) extends Sklar's theorem (1) to be applicable to time-series data.

$\hat{\theta}_2$:

$$\hat{\theta}_c = \arg\max_{\theta_c} \sum_{t=1}^{T} \log c(\hat{u}_{1t}, \hat{u}_{2t}|\Omega_{t-1}; \theta_c), \tag{10}$$

where $\hat{u}_{it} = F_i(x_{it}|\Omega_{t-1}; \hat{\theta}_i)$ for $i = 1, 2$. This estimation method has the benefit of being computationally tractable at the cost of a loss of full efficiency compared to the one-step ML method (Patton, 2009). In Sections 2.1 and 2.2, it is implicitly assumed that marginal models do not depend on unobserved states as in Chollete et al. (2009) and Garcia and Tsafack (2011). This assumption allows us to use the two-step estimation procedure since the state variable s_t is involved only in the log-likelihood for copula. Although the assumption seems to be restrictive, notice that conditional mean and variance of the marginal distributions are allowed to vary over time.

Estimation of the regime-switching copula model requires a further step to evaluate the log-likelihood for copula. Since our copula function is state-dependent as defined by Eq.(3), the copula density in Eq.(8) is given by

$$\begin{aligned} c(u_{1t}, u_{2t}|\Omega_{t-1}; \theta_c) &= \Pr(s_t = 1|\Omega_{t-1}; \theta_c) \times c_1(u_{1t}, u_{2t}|s_t = 1, \Omega_{t-1}; \theta_c) \\ &\quad + \Pr(s_t = 2|\Omega_{t-1}; \theta_c) \times c_2(u_{1t}, u_{2t}|s_t = 2, \Omega_{t-1}; \theta_c) \\ &= \xi'_{t|t-1} \cdot \eta_t, \end{aligned} \tag{11}$$

where $\xi_{t|t-1} = [\Pr(s_t = 1|\Omega_{t-1}; \theta_c), \Pr(s_t = 2|\Omega_{t-1}; \theta_c)]'$, $\eta_t = [c_1(u_{1t}, u_{2t}|s_t = 1, \Omega_{t-1}; \theta_c)$, $c_2(u_{1t}, u_{2t}|s_t = 2, \Omega_{t-1}; \theta_c)]'$, and c_1 and c_2 are copula densities under state 1 and state 2, respectively. We use Hamilton's filter proposed by Hamilton (1994, Chap. 22) to calculate $\xi_{t|t-1}$ for $t = 1, ..., T$:

$$\xi_{t|t} = \frac{\xi_{t|t-1} \odot \eta_t}{\xi'_{t|t-1} \cdot \eta_t}, \tag{12}$$

$$\xi_{t+1|t} = P \cdot \xi_{t|t}, \tag{13}$$

where the symbol $\odot$ denotes element-by-element multiplication and P the transition probability matrix given by Eq.(4). The maximization (10) is implemented with Eqs.(11)–(13) given observations of probability integral transforms, $\hat{u}_{1t}$ and $\hat{u}_{2t}$, and a starting value $\xi_{1|0}$.[7]

After the estimation, we calculate the smoothed probabilities $\xi_{t|T}$ for $t = 1, ..., T$ by iterating on the following equation backwards:

$$\xi_{t|T} = \xi_{t|t} \odot \{P' \cdot [\xi_{t+1|T}(\div)\xi_{t+1|t}]\}, \tag{14}$$

where the sign $(\div)$ denotes element-by-element division and the ith element of $\xi_{t|T}$ represents the inference about the probability of being in state i at time t based on the full sample of data.

Using the smoothed probabilities, we can also compute the smoothed Kendall's τ rank correlation defined as

$$\tau_t = \Pr(s_t = 1|\Omega_T; \theta_c) \times \tau_1 + \Pr(s_t = 2|\Omega_T; \theta_c) \times \tau_2, \tag{15}$$

where $\Pr(s_t = i|\Omega_T; \theta_c)$ is the ith element of the smoothed probabilities $\xi_{t|T}$, and τ_i denotes the Kendall's τ rank correlation between two variables in state i, which is calculated from the copula parameter estimate (see Table 1). In this study, we interpret the estimates of the smoothed Kendall's τ (15) for $t = 1, ..., T$ as the time-series movement of the Kendall's τ rank correlation between stock and government bond returns. In addition, the smoothed Kendall's τ is exploited to identify safe haven government bonds. We explain this in detail in the next subsection.

[7] The starting value $\xi_{1|0}$ is set equal to the vector of unconditional probabilities for two states; that is, $[(1 - p_{22})/(2 - p_{11} - p_{22}), (1 - p_{11})/(2 - p_{11} - p_{22})]'$.

2.4 Statistical methods to identify safe haven government bonds

The main purpose of this paper is to examine whether government bonds under study qualify as safe haven assets against domestic and international stock markets. As a special case of the general definition of a safe haven asset given by Baur and Lucey (2010) and Baur and McDermott (2010), this study defines a safe haven government bond against a stock market as a government bond whose returns are negatively correlated with the stock returns in times of market stress or turmoil.[8] To identify safe haven government bonds after the estimation of the regime-switching copula model, we propose two different methods that are consistent with the safe haven definition presented above but that emphasize different aspects of safe haven properties.

The first method relies on the tail dependence estimate. Chui and Yang (2012) show that if a copula is rotated by 90 degrees clockwise to describe a negative dependence structure, its lower–upper and upper–lower tail dependence coefficients are equal to, respectively, the lower–lower and upper–upper tail dependence of the original copula (the latter two concepts are simply called the lower and upper tail dependence in Section 2.1 and in the usual copula literature). In other words, if we transform the variable u_2 into $1-u_2$ with u_1 kept unchanged, estimate a copula function using the two variables, and calculate the lower–lower (upper–upper) tail dependence coefficient, then we automatically obtain the lower–upper (upper–lower) tail dependence. In our analysis, if the lower–upper tail dependence is statistically significant, we regard it as evidence of the presence of a safe haven government bond because the lower–upper tail dependence indicates the joint probability of observing extreme negative values of stock returns and extreme positive values of government bond returns, which is in line with the definition of a safe haven government bond presented above.

The second method to identify safe haven government bonds uses a regression equation

[8] Baur and Lucey (2010) and Baur and McDermott (2010) provide the definition of a safe haven for a general asset class rather than for a specific asset class such as government bonds. Baur and McDermott (2010) define a "strong" ("weak") safe haven asset as an asset that is negatively correlated (uncorrelated) with another asset or portfolio in times of market stress or turmoil. Our study focuses on the strong safe haven because of tractability in the identification of safe haven government bonds.

of the smoothed Kendall's τ on stock market volatility. Specifically, the following equation is estimated by OLS:

$$\hat{\tau}_t = b_0 + b_1\hat{\sigma}_{1t} + b_2\hat{\sigma}_{1t}^2 + v_t, \tag{16}$$

where $\hat{\tau}_t$ denotes the estimate of the smoothed Kendall's τ computed from Eq.(15) and $\hat{\sigma}_{1t}$ the estimate of stock market volatility obtained from the GARCH model (7). To take into account possible non-linearity of the relationship between $\hat{\tau}_t$ and $\hat{\sigma}_{1t}$, the square of the stock market volatility is added to the regression equation. However, when this squared variable is not significant at the 5% level, the linear model is estimated instead of Eq.(16).

If the marginal effect of an increase in stock market volatility on the smoothed Kendall's τ is negative (i.e., $b_1 + 2b_2\sigma_{1t} < 0$ at some level of σ_{1t} in the quadratic equation or $b_1 < 0$ in the linear equation), it implies that as the stock market volatility increases marginally, Kendall's τ rank correlation between stock and government bond returns declines further. However, this indicates only the marginal effect at some point of the stock market volatility and, hence, is not evidence of a safe haven government bond. Instead, we determine a safe haven government bond if the "total effect" of increases in the stock market volatility from low (or normal) to high levels is negative; that is, $\int_l^h (b_1 + 2b_2\sigma_1)d\sigma_1 = b_1(h - l) + b_2(h^2 - l^2) < 0$ where l and h denote a low (or normal) and a high level of stock market volatility, respectively. This condition is justified by the fact that stock market volatility abruptly increases to high levels at the onset of a typical financial crisis as we show for our data set in the next section. Note, however, that this condition does not necessarily assure negative dependence between stock and government bond returns at volatile times; it only shows a negative change in the dependence. Thus, we need one more condition for the identification of safe haven government bonds; that is, the condition that at least one negative dependence regime is detected in the estimation of our regime-switching copula model.

The two methods introduced above can also be applied to the identification of contagion between stocks and government bonds. The cross-asset contagion is a concept opposite to flight-to-quality or safe haven, and it is defined in this study as a phenomenon by which stocks and government bonds are positively correlated in times of market stress or turmoil. In terms of tail dependence coefficient, we determine the existence of cross-asset contagion if the lower–lower tail dependence between stock and government bond returns is statistically significant. In terms of the regression equation (16), we determine the existence of cross-asset contagion if the condition, $\int_l^h (b_1 + 2b_2\sigma_1) d\sigma_1 > 0$, is satisfied and at least one positive dependence regime is found from the estimation of the regime-switching copula model.

Moreover, we test the hedge role in government bonds. Again, following the definition of hedge presented in Baur and Lucey (2010) and Baur and McDermott (2010), we state that a government bond serves as a hedge against a stock market if returns on the government bond are negatively correlated with returns on the stock market *on average*. This definition is not confined to financially volatile times and requires assessing the average relationship over the entire sample period. Hence, the test is implemented by estimating the following equation:

$$\hat{\tau}_t = b_0 + v_t. \tag{17}$$

If $b_0 < 0$, we qualify a government bond under study as a hedge because this condition implies that the Kendall's τ rank correlation between stock and government bond returns is negative, on average, over the sample period regardless of the level of stock market volatility.

3 Data and results

3.1 Data

In this section, we examine the existence of safe haven government bonds for the following eight developed countries: Australia, Canada, France, Germany, Japan, Switzerland, the United Kingdom, and the United States. Our analysis consists of two parts: the first part is an analysis on the within-country stock–bond return relationships to identify safe haven government bonds against domestic stock markets. The second part is an analysis on the relationships between returns on internationally diversified stock portfolios and the government bond returns of each country to identify safe haven government bonds against international stock markets. Our data are Wednesday-to-Wednesday weekly frequency spanning from January 13, 1999 to June 27, 2018 (1,016 observations for each return series), and all of the data are obtained from Datastream.

In the first part of the analysis, we use the MSCI stock price indices and the Datastream 10-year government bond price indices for the eight countries, both of which are expressed in their own local currencies. The return series are calculated as 100 times the log-difference of the respective price indices.

In the second part of the analysis, we use the MSCI World stock price index (expressed in US dollars) as a proxy for the value of internationally diversified stock portfolios. According to MCSI, the price index is designed to represent the performance of large and mid-cap stocks across 23 developed markets covering approximately 85% of the free float-adjusted market capitalization in each country. Henceforth, we call the log-difference return of the MSCI World price index "world stock return." In our data set, world stock return is highly correlated with each of the eight countries' stock returns used in the first analysis (the linear correlation is 0.70 for Australia, 0.80 for Canada, 0.85 for France, 0.84 for Germany, 0.65 for Japan, 0.77 for Switzerland, 0.85 for the United Kingdom, and 0.94 for the United States). For government bond returns of the eight countries, we use the same unit of account as world stock return, that is, the US dollar rather than other respective local currencies. This is calculated as the sum of the government bond return whose price index is denominated in local currency, which is used in the first analysis, and the currency return, which is the log-difference of the WM/Reuters exchange rate in terms of US dollars per unit of the local currency.

The use of weekly frequency data is appropriate for our study because it avoids problems associated with large noises that would be present in daily or higher-frequency data. Moreover, the use of weekly frequency data avoids problems due to time zone differences across countries which should be considered in the second analysis where international linkages between stock and government bond markets are examined. The starting period of our sample (January 13, 1999) is selected to coincide with the introduction of the euro to financial markets since the euro is needed in the second analysis. Our sample period is suitable for detecting safe haven government bonds because it contains major financial crisis episodes such as the dot-com bubble crash in the early 2000s, the 9/11 attacks in 2001, the Iraq War from 2003, the global financial crisis from the summer of 2007, and the European debt crisis from late 2009.

[Table 2 around here]

Panels A and B of Table 2 present descriptive statistics of the stock and government bond returns, respectively, which are all denominated in local currencies (including the world stock returns in Panel A). The sample mean of stock returns is not necessarily greater than that of government bond returns: the government bonds of three countries (France, Germany, and the United Kingdom) yield higher returns than their corresponding stock returns, and the stock and government bond returns are nearly equal in Switzerland. This is probably because our sample period includes times of two devastating episodes, the global financial crisis and the European debt crisis, which had substantial negative effects on stock markets but might have had positive effects on government bond prices through flight-to-quality behavior. On the other hand, standard deviations of the stock and government bond returns show a conventional characteristic of higher risk in stocks relative to bonds. In addition, the two return series in all of the countries have negative skewness (other than UK bond returns) and excess kurtosis (greater than three), indicating a longer left tail and fatter tails compared to the normal distribution. This is statistically confirmed by the Jarque–Bera test reported in the last column in which the null hypothesis of normality is rejected for all returns. Therefore, it is justifiable to use Student's t and skewed t distributions in addition to normal distribution when modeling conditional marginal distributions of the stock and government bond returns.

Panels C and D of Table 2 provide descriptive statistics of the government bond returns expressed in US dollars and the currency returns in terms of US dollars per unit of local currency, respectively. Recall that government bond returns in US dollars (Panel C) are constructed by summing government bond returns in local currencies (Panel B) and currency returns (Panel D). Note that standard deviations of the government bond returns in US dollars reported in Panel C are greater than those in local currencies shown in Panel B because currency returns exhibit relatively high volatility as seen from Panel D. Again, skewness, kurtosis, and Jarque–Bera test statistics imply that the use of normal distribution might not be appropriate for modeling conditional marginal distributions of the returns in Panels C and D, supporting the additional use of Student's t and skewed t distributions.[9]

3.2 Results for marginal models

The first step in a copula approach requires estimating univariate marginal models for individual data. In this paper, the following procedure is taken for each return series to determine its best marginal model. First, we estimate several ARMA(p,q)–GJR–GARCH(1,1) models (5)–(7) with p and q up to 1, which have different specifications in terms of conditional mean, conditional variance, and error distribution. Specifically, we consider four conditional mean models (ARMA(0,0), ARMA(1,0), ARMA(0,1), and ARMA(1,1)); two conditional variance models (GARCH(1,1) with and without a GJR term); and three distributions for the standardized disturbance (standard normal, Student's t, and skewed t distributions). As a result, a total of 24 marginal models are estimated for each return series. Second, among them, a candidate for the best marginal model is chosen based on AIC. Third, for that candidate, the following four diagnostic tests are implemented: the Ljung–Box serial correlation test applied to standardized residuals $\hat{z}_{it}$, the same test applied to squared standardized residuals $\hat{z}_{it}^2$ (the lag order of the two tests is set to 12 weeks), and the Kolmogorov–Smirnov and Anderson–Darling goodness-of-fit tests applied to the probability integral transforms of the standardized residuals $\hat{u}_{it}$. If the candidate model is well specified, $\hat{z}_{it}$ and $\hat{z}_{it}^2$ would exhibit no serial correlation and $\hat{u}_{it}$ would be generated from an i.i.d. uniform $(0, 1)$ distribution.

[9] The currency returns are directly used as the input of the copula model in the decomposition analysis of Section 3.5.

Thus, if the candidate model passes all of the four tests at the 5% significance level, we regard it as the best marginal model that is correctly specified. If it fails any of the four tests, we return to and repeat the second and third steps until we obtain the best marginal model.

[Table 3 around here]

Table 3 reports the estimation results of the best marginal models for all of the return data used in this study.[10] Panel A shows that the stock returns of all countries (other than Australia) and world stock returns are specified by the skewed t distribution with a degree of freedom parameter that is statistically significant and a skewness parameter that is significantly negative. This means that the marginal distributions of these stock returns have fat tails and a longer left tail. The exception is the stock return of Australia where Student's t distribution rather than skewed t distribution is selected as the best marginal model, but the degree of freedom parameter is estimated to be large and insignificant. The coefficient on the GJR term is significantly positive for stock returns of all countries (other than Canada) and for world stock returns, suggesting that the leverage effect is present in these stock markets.

The marginal models of government bond returns have slightly different features from stock returns. Panels B and C show that the best marginal model for each of the government bond returns is either Student's t or skewed t distribution (except for US government bond returns in Panel B for which normal distribution is selected as the best marginal model). The estimated degree of freedom parameters of the Student's t or skewed t distribution are all statistically significant (except for Germany in Panel B), implying that they have fat tails as with stock returns. However, a significantly negative estimate of the skewness parameter is only limited to three countries in Panel B and two countries in Panel C. Moreover, all of the estimates for the GJR coefficient are insignificant. These results provide weak or no evidence on the presence of a longer left tail and the leverage effect for the government bond returns.

Panel D shows that the best marginal models for currency returns have similar characteristics to government bond returns: they have fat tails (except for the Canadian dollar) but

[10]To save space, we do not report the estimates of ARMA and GARCH coefficients as they are relatively less informative. These results are available from the authors upon request.

present mixed evidence on a longer left tail and no evidence on the leverage effect.

Finally, from p-values of four diagnostic tests reported in the last four columns, we confirm that all the marginal models are correctly specified, although this is obvious from our procedure of selecting the best marginal models.

[Figure 1 around here]

Figure 1 displays the evolution of volatilities of individual stock returns and world stock returns, $\hat{\sigma}_{1t}$, which is calculated from the estimated GARCH models (and recall that the volatilities are used in estimating Eq.(16)). We confirm that periods of high stock market volatility correspond to the main crisis episodes (the dot-com bubble crash in the early 2000s, the 9/11 attacks in 2001, the Iraq War from 2003, the global financial crisis from the summer of 2007, and the European debt crisis from late 2009) and hence, the volatility movements are very similar across countries. However, a closer look at the figure finds some differences. First, stock markets in France, Germany, and Switzerland are even more volatile during the European debt crisis compared to the other countries. Second, surprisingly, the stock market volatilities of four European countries in the early 2000s are greater than the volatility of the US stock market even though three crisis episodes in this period (the dot-com bubble crash, the 9/11 attacks, and the Iraq War) are more closely related to the United States. Third, stock markets are relatively stable during the early 2000s in Australia and during the European debt crisis in Australia and Canada.

3.3 Identifying safe haven government bonds against domestic stock markets

Given the observations for the probability integral transformations of standardized residuals, $\{\hat{u}_{it}\}_{t=1}^{T}$ for $i = 1, 2$, we next estimate the two-state regime-switching copula model (3)–(4). In particular, in this subsection we examine the within-country dependence structures between stock and government bond returns and determine safe haven government bonds against domestic stock markets.

[Table 4 around here]

Table 4 reports estimation results on the domestic stock–bond return relations for eight developed countries, which include dependence signs, best copula functions selected, estimated parameters of the copula functions, implied estimates of Kendall's τ and four tail dependence coefficients computed from the copula parameter estimates,[11] and the estimated transition probabilities. We find that all countries have at least one negative dependence regime with a negative and statistically significant estimate of Kendall's τ. This suggests the possibility that government bonds of all of the countries might have been safe haven assets against their own domestic stock markets since the condition of a negative value of Kendall's τ is needed for a government bond to be a safe haven when the identification relies on the second method (a regression of smoothed Kendall's τ). Moreover, positive–negative dependence regimes are selected for five countries, and the corresponding two copula functions are not the combination of two Gaussian (or two Student's t) copulas. These results indicate the benefit of using our two-state regime-switching copula model rather than ARMA-type time-varying specifications with which only positive (or negative) dependence can be described by a fixed copula function, except for some copulas such as Gaussian or Student's t copula.

[Figures 2 & 3 around here]

Figure 2 plots the movement of the smoothed probabilities calculated from Eq.(14) for a lower dependence regime (namely, a negative dependence regime for the country with positive–negative dependence regimes and a lower negative dependence regime for the country with negative–negative dependence regimes). The negative or lower negative dependence regime frequently occurs with high probabilities during the early 2000s and from 2007 to 2012, the periods corresponding to the occurrence of major financial crises. The similar pattern is also seen in Figure 3, which displays the time-series estimates of the smoothed Kendall's τ obtained through Eq.(15). That is, the Kendall's τ rank correlation between domestic stock and government bond returns appears to decline particularly at times of major financial crises.

[11] The delta method is used to compute standard errors for the Kendall's τ and tail dependence coefficients.

These findings might support the existence of safe haven government bonds, but we need to provide more rigorous evidence by using two statistical methods proposed in Section 2.4. First, in Table 4, the lower–upper tail dependence (denoted as "LU tail" in the table) is statistically significant for Japan and Switzerland, suggesting that these two countries have positive probability of jointly observing extreme negative returns on stocks and extreme positive returns on government bonds. Thus, we state that Japanese and Swiss government bonds have served as safe haven assets against their own domestic stock markets, although evidence on the Japanese government bond is weak as its statistical significance level is only 10%.

[Table 5 around here]

Second, employing an alternative method to identify safe haven government bonds leads to different results. Table 5 reports the results of estimating Eq.(16) in which the smoothed Kendall's τ is regressed on the stock market volatility and its squared variable (if it is statistically significant at the 5% level).[12] The coefficient on the stock market volatility, b_1, is significantly negative for all countries other than Japan. On the other hand, the coefficient on the squared volatility, b_2, is significantly positive in Australia, Canada, the United Kingdom, and the United States; however, their estimated values are much smaller than those of b_1. These results imply that a marginal increase in the stock market volatility at low levels decreases the degree of the dependence between domestic stocks and government bonds, and the marginal effect is constant or diminishing as the volatility reaches high levels.

As discussed in Section 2.4, to identify safe haven government bonds using the second method, we must determine the sign and statistical significance of the total effect which is defined as an accumulative effect of increases in stock market volatility from low (or normal) to high levels on the stock–bond return correlation; that is, $\int_l^h (b_1 + 2b_2\sigma_1)d\sigma_1 = b_1(h-l) + b_2(h^2 - l^2)$ where l and h denote a low (or normal) and a high level of stock market volatility, respectively.[13] As Figure 1 shows, financial crisis episodes are characterized by large abrupt

[12]To take into account the autocorrelation of the error terms in Eqs.(16) and (17), we use Newey–West heteroscedasticity and autocorrelation consistent standard errors for inference.

[13]The delta method is used to compute standard errors of the total effects.

increases in stock market volatility. Thus, the total effect represents the difference between the stock–bond return correlations under normal and financial crisis situations and, if it is significantly negative with at least one negative dependence regime, it is consistent with the definition of a safe haven government bond. Since a choice of the values of l and h is arbitrary, we set l equal to either the 25th or 50th percentile of stock market volatility and h equal to either the 90th or 95th percentile, which leads to a consideration of four sets of (l, h).

Table 5 shows the estimates of the total effects. In six countries (Australia, France, Germany, Switzerland, the United Kingdom, and the United States), the total effects are negative and statistically significant in all four sets of (l, h), suggesting that the government bonds of these six countries have served as safe haven assets against their own domestic stock markets. The total effects for Switzerland, however, are very small ranging from -0.018 to -0.031. For the other countries, they are relatively widely distributed from -0.042 to -0.154.

In summary, in terms of the lower–upper tail dependence, Japanese and Swiss government bonds are shown to be safe havens (but the evidence for Japan is weak). Meanwhile, in terms of the regression of smoothed Kendall's τ, the government bonds of six countries (Australia, France, Germany, Switzerland, the United Kingdom, and the United States) are regarded as safe havens. Therefore, by relying on the result from one of the two methods or both, we find that all of the government bonds (except for Canadian government bonds) qualify as safe havens against their own domestic stock markets. The evidence on the existence of safe haven government bonds against domestic stock markets (or flights-to-quality from stocks to bonds within countries) for several developed countries has been documented by many studies (e.g., Andersson et al., 2008; Baur and Lucey, 2009; Chen et al., 2014; Chiang et al., 2015; Chui and Yang, 2012; Ciner et al., 2013; Connolly et al., 2007; Hartmann et al., 2004; Jammazi et al., 2015; Kim et al., 2006; Nguyen and Liu, 2017; Ohmi and Okimoto, 2016). Our results reinforce their findings by using the two-state regime-switching copula model and testing safe haven properties with the two statistical methods.

It is also worth mentioning the results regarding contagion and hedges. With regard to contagion, we find that there has been no contagion across domestic stocks and government bonds over the sample period. This is supported by the following results:(i) as shown in Table 4, no country has a lower–lower tail dependence coefficient (denoted as "LL tail" in the table) that is statistically significant;(ii) as seen from Table 5, there are no countries for which the total effects of increases in stock market volatility on the stock–bond return correlation are significantly positive. This finding contradicts a finding of Hartmann et al. (2004) in which stock–bond contagion is approximately as frequent as flight-to-quality from stocks to bonds in the G-5 countries for the period 1987 to 1999. The difference between their result and ours might stem from the difference in the sample period. In fact, examining the more recent period 1994 to 2006 for eight developed countries, Baur and Lucey (2009) show that stock–bond contagion only exists during the crisis period of the 9/11 attacks in three countries, whereas they provide evidence on more frequent occurrence of flights (flight-to-quality from stocks to bonds and flight-from-quality from bonds to stocks). With regard to a hedge role of government bonds, we find that all of the eight government bonds have served as hedges against domestic stock markets. This is confirmed from the last column of Table 5 in which the constant term of Eq.(17) is estimated to be negative in all countries, implying that their government bond returns are, on average, negatively correlated with domestic stock returns over the sample period.

3.4 Identifying safe haven government bonds against international stock markets

We next analyze the dependence structure between world stock returns and individual government bond returns and determine safe haven government bonds against international stock markets. The price indices used in this subsection for the calculation of the world stock returns and government bond returns are all denominated in US dollars. Hence, the purpose of the analysis is to identify safe haven government bonds from the standpoint of US residents who hold internationally diversified stock portfolios. Tables and figures corresponding to those in the previous subsection are also presented here: Table 6 reports estimation results for the two-state regime-switching copula model; Figure 4 plots the movements of the smoothed probabilities for a lower dependence regime; Figure 5 displays the movements of the smoothed Kendall's τ rank correlation between world stock returns and individual government bond returns; and Table 7 provides the test results of safe haven government bonds based on the regression of the smoothed Kendall's τ.

[Tables 6 & 7 around here]

[Figures 4 & 5 around here]

The results from these tables and figures are remarkably different from the results on the domestic stock–bond return relations reported in the previous subsection. First, Table 6 shows that the relationship between world stock returns and each of the Australian and Canadian government bond returns is characterized as a combination of positive–positive dependence regimes, meaning that the government bonds of the two countries are no longer safe haven assets against international stock markets. This is in contrast to the corresponding result in the previous subsection where the two countries have at least one negative dependence regime and Australian government bonds in particular qualify as a safe haven against the domestic stock market.

Second, comparing Figure 5 to Figure 3 reveals that the evolution of the smoothed Kendall's τ distinctly differs for each country except for the United States: the movements of the smoothed Kendall's τ shown in Figure 5 are more persistent and have higher values than those in Figure 3. In the next subsection, we discuss the reason why such differences arise by conducting a decomposition analysis.

Third, we find a smaller set of safe haven government bonds compared to the set in the previous subsection. With regard to the first identification method, the estimates of the lower–upper tail dependence reported in Table 6 show that only Japanese government bonds qualify as a safe haven against international stock markets, whereas in the previous subsection both Japanese and Swiss government bonds have shown to be safe havens against domestic stock markets. With regard to the second identification method, Table 7 shows that the total effects of increases in stock market volatility from low (or normal) to high levels on the smoothed Kendall's τ are negative and statistically significant in all four sets of (l, h) for Germany, Japan, the United Kingdom, and the United States, suggesting that the government bonds of the four countries can be regarded as safe havens against international stock markets. On the other hand, in the previous analysis, the government bonds of six countries (Australia, France, Germany, Switzerland, the United Kingdom, and the United States) have qualified as safe havens against domestic stock markets.

Fourth, related to the third difference above, the safe haven roles of German, Japanese, and UK government bonds against international stock markets were played during only limited periods and shifted across countries over time. Figures 4 and 5 show that a negative dependence regime in the United Kingdom occurred only in the early 2000s and the negative dependence regime in Germany was dominant in the early 2000s and in some short periods between 2007 and 2016. Japanese government bond returns, on the other hand, were negatively correlated with the world stock returns throughout the period 2007 to 2016. These results imply that the government bonds of the United Kingdom and Germany served as safe havens against international stock markets in the early 2000s and Japanese government bonds had played that role from the outset of the global financial crisis in 2007. This might be explained by the fact that the global financial crisis and the subsequent European debt crisis had less direct effects on the Japanese economy than on the German and UK economies. In the next subsection, we document evidence on the source of the shifts in the safe haven role.

In contrast to the transitory safe haven role of the three countries' government bonds, US government bonds have served as a safe haven against international stock markets over the whole sample period. The smoothed Kendall's τ displayed in the US panel of Figure 5 takes lower negative values (around -0.4) many times in the sample period. Additionally, the total effects for the United States reported in Table 7 are all negative and significant, implying that US government bonds have always played a safe haven role during financial crises. Note also that the evolution of the smoothed Kendall's τ for the United States is similar to the evolution reported in Figure 3 because of a high correlation (0.94) between the US and world stock returns.

Fifth, related to the first difference above, there is evidence on cross-asset contagion between international stock markets and Australian and Canadian government bonds. The total effects for the two countries reported in Table 7 are all positive and significant, supporting the presence of the cross-asset contagion. This is in contrast to the evidence from Table 5 that there has been no cross-asset contagion between domestic stock and government bond markets. Moreover, Figure 5 shows that the smoothed Kendall's τ between world stock returns and each of the Australian and Canadian government bond returns increased with the bankruptcy of Lehman Brothers in September 2008, suggesting that the cross-asset contagion occurred in this period.

Finally, as the last column of Table 7 shows, only Japanese and US government bonds qualify as hedges against international stock markets. This exhibits a remarkable difference from the result in Table 5 in which the government bonds of all eight countries have been shown as hedges against domestic stock markets.

3.5 Decomposition analysis

We have found several differences between the relations of domestic stock and government bond returns (Section 3.3) and the relations of world stock returns and individual government bond returns (Section 3.4). Why do such differences arise? To answer this question, we decompose each of the latter relations into two sub-relations: the first is between world stock returns and individual government bond returns where the price indices used in the calculation of the government bond returns are denominated in local currencies instead of US dollars. The second is between world stock returns and individual currency returns whose corresponding exchange rates are expressed in terms of US dollars per unit of local currency. For each of the two sub-relations, we estimate the same regime-switching copula model as before.

[Figures 6 & 7 around here]

To save space, only the results on the smoothed Kendall's τ are reported, but this is sufficient to answer the question raised above.[14] Figures 6 and 7 display the estimates of the smoothed Kendall's τ for the first and second sub-relations, respectively. Notice that the movements of the smoothed Kendall's τ shown in Figure 7 more closely resemble those in Figure 5 with common features of persistent dynamics and relatively high values. This suggests that the second sub-relation affects the empirical results of Section 3.4 more strongly than the first sub-relation. In other words, the decomposition analysis shows that the way in which local currencies are correlated with international stock markets is an important factor in determining the co-movements between world stock returns and government bond returns whose price indices are both denominated in US dollars.

As a result, the currency return dynamics dominantly affect the determination of safe haven government bonds. In the previous subsection, we found that the government bonds of Germany, Japan, and the United Kingdom (and the United States) served as safe havens against international stock markets. However, we also found that the safe haven role shifted from German and UK government bonds in the early 2000s to Japanese government bonds in the period 2007 to 2016. The decomposition analysis conducted here implies that the shift of the safe haven role is explained by changes in the status of the three countries' currencies in the global financial markets. The euro and UK pound acquired safe haven status during the early 2000s and, thus, tended to appreciate in response to financial crisis shocks (which is indicated by negative correlations between world stock returns and the returns on the euro or UK pound during the early 2000s in Figure 7). In contrast, the Japanese yen played a safe haven role during the period 2007 to 2016 (which is also indicated by negative correlations between world stock returns and the returns on the Japanese yen for that period). Tachibana (2018) documents evidence that the Japanese yen has become the most important currency both as a safe haven and as a hedge against the US stock market since the 2007 global

[14] All estimation results are available from the authors upon request.

financial crisis. Our finding is consistent with this evidence because US stock returns are highly correlated with the world stock returns in our data set.

Similarly, although in the previous subsection we found cross-asset contagion between international stock markets and Australian and Canadian government bonds during the bankruptcy of Lehman Brothers in September 2008, Figure 7 shows that this evidence is attributable to the depreciation in Canadian and Australian dollars against the US dollar at that time (which is indicated by increases in the smoothed Kendall's τ for the two countries at the time). Finally, the smoothed Kendall's τ plotted in Figure 7 is positive throughout most of the sample period for all of the currencies except for the Japanese yen, implying that these currencies are, on average, positively correlated with international stock markets. This explains the finding in the previous subsection that only Japanese and US government bonds qualify as hedges against world stock returns.

4 Conclusion

This paper examines the dynamic relationship between stock and government bond returns and identifies safe haven government bonds against domestic and international stock markets. We develop a two-state regime-switching copula model that can describe various possible patterns of the stock–bond return dependence structure. We apply this model to pairs of domestic stock and government bond returns of eight developed countries as well as to the pairs of world stock returns and individual government bond returns over the period 1999 to 2018. We then determine safe haven government bonds by relying on two different statistical methods.

Our main findings are summarized as follows. First, the government bonds of all of the countries (except for Canada) qualify as safe havens against their own domestic stock markets. Japanese and Swiss government bonds are shown to be safe havens from the estimated lower–upper tail dependence while the government bonds of six countries (Australia, France, Germany, Switzerland, the United Kingdom, and the United States) are shown to be safe havens based on the regression of smoothed Kendall's τ. Second, we find a smaller set of safe haven government bonds against international stock markets than against domestic stock markets, and the contents of the set vary over time. Only US government bonds have served as a safe haven against international stock markets over the entire sample period. On the other hand, German and UK government bonds played the safe haven role mainly at crisis times in the early 2000s, whereas the role shifted to Japanese government bonds for the period 2007 to 2016, which includes episodes of the global financial crisis and the European debt crisis. Third, the decomposition analysis shows that how local currencies are correlated with international stock markets is an important factor in determining safe haven government bonds against international stock markets. Thus, the shift in the safe haven role from German and UK government bonds to Japanese government bonds since the onset of the 2007 global financial crisis is explained by the shift in the safe haven status of their currencies in the global financial markets.

The empirical results of this paper have practical implications for asset allocation and risk management. The results imply that holding stocks and government bonds together in a developed country where investors live would help to protect portfolio values against large declines in stock prices. However, investing in international stock and government bond markets requires caution in the selection of government bonds to diversify portfolio risk because when a financial crisis occurs, the prices of the foreign government bonds purchased might collapse due to depreciation of the foreign currencies. In this paper, we identify safe haven government bonds against international stock markets only from the standpoint of US investors. In future research, to derive more practical implications, it would be fruitful to explore the subject from the viewpoint of investors who live in other countries.

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Table 1: Copula functions

Name	Copula CDF	Kendall's τ	Lower tail	Upper tail
Gaussian	$C_N(u_1, u_2; \rho) = \Phi_2\left(\Phi^{-1}(u_1), \Phi^{-1}(u_2); \rho\right)$	$2\pi^{-1}\arcsin(\rho)$	0	0
Student's t	$C_T(u_1, u_2; \rho, \nu) = T_{2,\nu}\left(T_\nu^{-1}(u_1), T_\nu^{-1}(u_2); \rho\right)$	$2\pi^{-1}\arcsin(\rho)$	See notes below	
Clayton	$C_C(u_1, u_2; \delta) = (u_1^{-\delta} + u_2^{-\delta} - 1)^{-1/\delta}$	$\delta/(\delta+2)$	$2^{-1/\delta}$	0
Gumbel	$C_G(u_1, u_2; \delta) = \exp\left(-\left[(-\log u_1)^\delta + (-\log u_2)^\delta\right]^{1/\delta}\right)$	$(\delta-1)/\delta$	0	$2-2^{1/\delta}$
BB7	$C_{BB7}(u_1, u_2; \delta, \theta) = 1 - \left(1 - \left[(1-(1-u_1)^\theta)^{-\delta} + (1-(1-u_2)^\theta)^{-\delta} - 1\right]^{-1/\delta}\right)^{1/\theta}$	See notes below	$2^{-1/\delta}$	$2-2^{1/\theta}$

Notes: The table presents cumulative distribution functions (CDFs) of the bivariate copulas used for a positive dependence regime and corresponding functions of the Kendall's τ rank correlation and the lower and upper tail dependence. The parameter space for the linear correlation of Gaussian and Student's t copulas is restricted to be positive; that is, $\rho \in (0, 1]$. Φ and Φ_2 (T_ν and $T_{2,\nu}$) denote the CDFs of the standard univariate and bivariate normal (Student's t) distributions, respectively. The Kendall's τ of BB7 copula is: $\tau = 1 - 2\delta^{-1}(2-\theta)^{-1} + 4\delta^{-1}\theta^{-2}B(\delta+2, 2/\theta - 1)$ where $B(\cdot,\cdot)$ denotes the beta function. The tail dependence of Student's t copula is: $\lambda_L = \lambda_U = 2T_{\nu+1}\left(-\sqrt{(\nu+1)(1-\rho)/(1+\rho)}\right)$. For a negative dependence regime, we use 90-degree clockwise rotations of the five copulas in the table, which are defined as $C_{90}(u_1, u_2) = u_1 - C(u_1, 1-u_2)$. The Kendall's τ rank correlation is multiplied by -1 in the case of a negative dependence regime.

Table 2: Descriptive statistics

	Mean	Max	Min	SD	Skewness	Kurtosis	JB
Panel A: Stock returns in local currencies							
Australia	0.075	11.604	-11.988	2.051	-0.431	6.115	442.28 ***
Canada	0.095	8.273	-14.379	2.284	-0.612	6.097	469.39 ***
France	0.035	16.550	-15.002	2.954	-0.354	6.580	563.84 ***
Germany	0.031	16.461	-16.493	3.181	-0.665	6.646	637.52 ***
Japan	0.041	15.542	-20.482	2.974	-0.484	6.694	617.16 ***
Switzerland	0.019	14.571	-13.852	2.492	-0.517	8.280	1225.57 ***
UK	0.019	13.344	-12.437	2.357	-0.330	6.604	568.26 ***
US	0.072	10.344	-16.748	2.325	-0.668	8.137	1192.61 ***
World	0.054	9.268	-16.546	2.293	-0.812	7.637	1021.97 ***
Panel B: Government bond returns in local currencies							
Australia	0.016	3.885	-4.061	0.953	-0.233	3.998	51.39 ***
Canada	0.030	3.206	-3.185	0.791	-0.143	3.592	18.28 ***
France	0.041	2.189	-4.118	0.785	-0.495	4.566	145.27 ***
Germany	0.039	2.534	-3.153	0.791	-0.252	3.774	36.10 ***
Japan	0.032	2.155	-4.510	0.536	-1.437	12.590	4242.80 ***
Switzerland	0.019	3.383	-3.864	0.677	-0.216	5.106	195.62 ***
UK	0.031	4.401	-3.310	0.866	0.106	4.282	71.43 ***
US	0.014	4.052	-4.176	1.018	-0.050	4.039	46.12 ***
Panel C: Government bond returns in US dollars							
Australia	0.032	7.976	-13.163	1.749	-0.619	6.889	705.18 ***
Canada	0.043	6.384	-6.453	1.301	-0.113	5.035	177.46 ***
France	0.040	11.384	-6.994	1.587	0.176	5.683	310.02 ***
Germany	0.038	11.667	-6.967	1.543	0.282	6.245	459.15 ***
Japan	0.033	6.732	-7.194	1.553	0.068	4.556	103.33 ***
Switzerland	0.051	20.146	-8.232	1.762	1.524	21.676	15158.60 ***
UK	0.008	6.574	-8.153	1.435	-0.386	5.266	242.59 ***
Panel D: Currency returns in terms of US dollars per unit of local currency							
Australia	0.016	6.355	-17.049	1.699	-1.295	14.059	5461.27 ***
Canada	0.013	6.071	-6.025	1.251	-0.106	5.458	257.74 ***
Euro	-0.001	9.809	-5.093	1.384	0.235	5.539	282.26 ***
Japan	0.002	6.180	-6.911	1.390	0.077	4.518	98.50 ***
Switzerland	0.032	16.763	-8.602	1.543	1.330	18.708	10744.85 ***
UK	-0.023	5.296	-8.243	1.321	-0.652	6.622	627.44 ***

Notes: Weekly data on stock returns, government bond returns, and currency returns for the period January 13, 1999 to June 27, 2018. The number of observations for each variable is 1,016. SD denotes the standard deviation. JB is the Jarque–Bera statistics for the test of normality. *** indicates rejection of the null hypothesis of normality at the 1% level.

Table 3: Estimation results for marginal models

	Dist.	p	q	GJR	DoF	Skewness	$Q(12)$	$Q^2(12)$	KS	AD
Panel A: Stock returns in local currencies										
Australia	T	1	1	0.190 ***	96.147		[0.803]	[0.988]	[0.067]	[0.174]
				(0.070)	(260.493)					
Canada	S	0	1	0.094	17.939 *	-0.240 ***	[0.429]	[0.673]	[0.513]	[0.840]
				(0.082)	(10.357)	(0.046)				
France	S	1	0	0.269 ***	9.332 ***	-0.276 ***	[0.904]	[0.581]	[0.688]	[0.783]
				(0.084)	(3.243)	(0.044)				
Germany	S	1	0	0.173 **	8.942 ***	-0.284 ***	[0.631]	[0.894]	[0.518]	[0.669]
				(0.078)	(2.582)	(0.046)				
Japan	S	1	1	0.196 **	7.148 ***	-0.170 ***	[0.303]	[0.988]	[0.910]	[0.900]
				(0.085)	(1.497)	(0.044)				
Switzerland	S	1	0	0.345 ***	8.481 ***	-0.322 ***	[0.979]	[0.965]	[0.320]	[0.673]
				(0.088)	(1.772)	(0.043)				
UK	S	1	0	0.326 ***	18.732 *	-0.332 ***	[0.943]	[0.455]	[0.992]	[0.992]
				(0.077)	(10.347)	(0.048)				
US	S	0	1	0.262 ***	7.837 ***	-0.332 ***	[0.760]	[0.823]	[0.972]	[0.983]
				(0.095)	(1.756)	(0.043)				
World	S	0	1	0.214 **	11.813 ***	-0.320 ***	[0.691]	[0.806]	[0.567]	[0.765]
				(0.091)	(3.994)	(0.046)				
Panel B: Government bond returns in local currencies										
Australia	T	1	1	-0.078	16.283 **		[0.992]	[0.224]	[0.959]	[0.979]
				(0.059)	(7.427)					
Canada	S	1	1		13.025 **	-0.064	[0.856]	[0.250]	[0.677]	[0.702]
					(5.568)	(0.050)				
France	S	0	0	-0.044	12.413 ***	-0.113 **	[0.638]	[0.663]	[0.963]	[0.986]
				(0.083)	(4.588)	(0.049)				
Germany	S	0	0	-0.046	24.863	-0.083 *	[0.566]	[0.297]	[0.743]	[0.897]
				(0.078)	(15.643)	(0.048)				
Japan	S	1	0	-0.105	4.870 ***	-0.144 ***	[0.513]	[0.977]	[0.981]	[0.989]
				(0.100)	(0.643)	(0.043)				
Switzerland	T	0	0	-0.035	9.838 ***		[0.453]	[0.434]	[0.846]	[0.802]
				(0.069)	(2.112)					
UK	T	0	1	-0.055	12.995 ***		[0.908]	[0.440]	[0.968]	[0.978]
				(0.066)	(4.560)					
US	N	1	1	-0.072			[0.917]	[0.155]	[0.933]	[0.789]
				(0.049)						
Panel C: Government bond returns in US dollars										
Australia	S	0	0		9.850 ***	-0.179 ***	[0.289]	[0.236]	[0.678]	[0.968]
					(2.817)	(0.044)				
Canada	T	0	0		14.896 **		[0.909]	[0.328]	[0.989]	[0.996]
					(6.414)					
France	S	0	0	0.037	11.334 ***	-0.097 **	[0.752]	[0.521]	[0.810]	[0.843]
				(0.081)	(2.336)	(0.046)				
Germany	S	0	0		11.201 ***	-0.071	[0.898]	[0.794]	[0.849]	[0.945]
					(2.443)	(0.046)				
Japan	T	1	1		8.266 ***		[0.947]	[0.362]	[0.896]	[0.947]
					(1.903)					
Switzerland	T	0	0	-0.095	6.635 ***		[0.857]	[1.000]	[0.849]	[0.890]
				(0.080)	(0.714)					
UK	T	1	1	0.003	10.224 ***		[0.182]	[0.798]	[0.186]	[0.179]
				(0.081)	(3.073)					

(Continued)

	Dist.	p	q	GJR	DoF	Skewness	$Q(12)$	$Q^2(12)$	KS	AD
Panel D: Currency returns in terms of US dollars per unit of local currency										
Australia	S	0	0	0.055	12.582 ***	-0.223 ***	[0.386]	[0.092]	[0.952]	[0.990]
				(0.069)	(4.758)	(0.046)				
Canada	N	0	0				[0.451]	[0.222]	[0.873]	[0.894]
Euro	T	0	0	0.069	23.217 *		[0.905]	[0.579]	[0.902]	[0.992]
				(0.075)	(12.842)					
Japan	T	1	1		9.717 ***		[0.403]	[0.451]	[0.809]	[0.857]
					(2.381)					
Switzerland	S	0	0		8.109 ***	0.087 *	[0.450]	[1.000]	[0.858]	[0.832]
					(0.982)	(0.049)				
UK	S	0	0	0.061	14.499 **	-0.101 **	[0.969]	[0.395]	[0.940]	[0.999]
				(0.063)	(6.116)	(0.049)				

Notes: The table reports estimation results on the best marginal models of stock returns, government bond returns, and currency returns, which are selected from the class of ARMA(p,q)–GJR–GARCH(1,1) models. The sample period is from January 13, 1999 to June 27, 2018. The first column (Dist.) shows selected distribution functions for the standardized disturbance where N, T, and S denote the standard normal, Student's t, and skewed t distributions, respectively. The second and third columns (p, q) present selected numbers of lag length for the AR and MA terms, respectively. The fourth to sixth columns (GJR, DoF, Skewness) report estimates of the coefficient on the GJR term, estimates of the degree of freedom parameter for the Student's t and skewed t distributions, and estimates of the skewness parameter for the skewed t distribution, respectively. Standard errors are reported in parentheses. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively. $Q(12)$ and $Q^2(12)$ denote the Ljung–Box serial correlation tests applied to standardized residuals and squared standardized residuals, respectively, with lag length of 12 weeks. KS and AD denote the Kolmogorov–Smirnov and Anderson–Darling tests, respectively, for the adequacy of marginal models. For these four tests, p-values are reported in brackets.

Table 4: Estimation results for two-state regime-switching copulas: relationships between domestic stock and government bond returns

	Dep. sign	Copula	Parameter		Kendall	LL tail	UU tail	LU tail	UL tail	Tran. prob.
Australia	Positive	Gaussian	0.247 **		0.159 **	0	0			0.973 ***
			(0.111)		(0.073)					(0.170)
	Negative	R. Student-t	0.390 ***	9.929	-0.255 ***			0.051	0.051	0.986 ***
			(0.072)	(6.334)	(0.050)			(0.056)	(0.056)	(0.128)
Canada	Negative	R. Gaussian	0.476 ***		-0.316 ***			0	0	0.968 **
			(0.145)		(0.105)					(0.398)
	Negative	R. Clayton	0.088		-0.042			0.000	0	0.983 ***
			(0.089)		(0.041)			(0.003)		(0.176)
France	Positive	Clayton	0.081		0.039	0.000	0			0.966 ***
			(0.079)		(0.037)	(0.002)				(0.126)
	Negative	R. Gaussian	0.533 ***		-0.358 ***			0	0	0.956 ***
			(0.097)		(0.073)					(0.208)
Germany	Positive	Gumbel	1.629 ***		0.386 ***	0	0.470 ***			0.791 ***
			(0.256)		(0.097)		(0.103)			(0.137)
	Negative	R. Gaussian	0.468 ***		-0.310 ***			0	0	0.949 ***
			(0.039)		(0.028)					(0.041)
Japan	Negative	R. Gaussian	0.059		-0.037			0	0	0.935 ***
			(0.080)		(0.051)					(0.126)
	Negative	R. Student-t	0.600 ***	7.775 *	-0.409 ***			0.173 *	0.173 *	0.937 ***
			(0.104)	(4.630)	(0.082)			(0.103)	(0.103)	(0.229)
Switzerland	Positive	Gaussian	0.671 ***		0.468 ***	0	0			0.772 ***
			(0.128)		(0.110)					(0.212)
	Negative	R. BB7	0.303 ***	1.159 ***	-0.194 ***			0.101 **	0.182 ***	0.967 ***
			(0.063)	(0.048)	(0.026)			(0.048)	(0.045)	(0.053)
UK	Positive	Clayton	0.076		0.037	0.000	0			0.971 ***
			(0.062)		(0.029)	(0.001)				(0.084)
	Negative	R. Gaussian	0.606 ***		-0.414 ***			0	0	0.962 ***
			(0.058)		(0.047)					(0.111)
US	Negative	R. Gaussian	0.664 ***		-0.463 ***			0	0	0.953 ***
			(0.067)		(0.057)					(0.169)
	Negative	R. Clayton	0.085		-0.041			0.000	0	0.969 ***
			(0.083)		(0.038)			(0.002)		(0.113)

Notes: The table reports the results from estimating the two-state regime-switching copula model for the relationship between domestic stock and government bond returns. The sample period is from January 20, 1999 to June 27, 2018. "R. copula name" denotes the 90-degree clockwise rotation of the copula. "Parameter" reports the estimates of copula parameters (see Table 1). LL, UU, LU, and UL tails denote the lower–lower, upper–upper, lower–upper, and upper–lower tail dependence coefficients, respectively. Transition probabilities reported in the last column correspond to the estimates of p_{11} and p_{22}. Standard errors are reported in parentheses. The delta method is used to compute standard errors for Kendall's τ and tail dependence coefficients. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.

Table 5: Test results for safe haven (and hedge) properties of government bond returns against domestic stock returns

	Test for safe haven							Test for hedge
	Eq.(16)			Total effect				Eq.(17)
	b_0	b_1	b_2	25th–90th	25th–95th	50th–90th	50th–95th	b_0
Australia	0.135 **	-0.174 ***	0.018 ***	-0.125 ***	-0.154 ***	-0.092 ***	-0.121 ***	-0.122 ***
	(0.053)	(0.035)	(0.005)	(0.024)	(0.026)	(0.018)	(0.020)	(0.013)
Canada	-0.072 **	-0.051 **	0.008 **	-0.020	-0.018	-0.011	-0.009	-0.137 ***
	(0.032)	(0.023)	(0.003)	(0.017)	(0.019)	(0.012)	(0.015)	(0.007)
France	-0.038 *	-0.035 ***		-0.085 ***	-0.119 ***	-0.067 ***	-0.102 ***	-0.134 ***
	(0.020)	(0.006)		(0.012)	(0.016)	(0.009)	(0.014)	(0.011)
Germany	-0.109 ***	-0.022 ***		-0.054 ***	-0.078 ***	-0.042 ***	-0.066 ***	-0.174 ***
	(0.027)	(0.007)		(0.014)	(0.019)	(0.011)	(0.016)	(0.013)
Japan	-0.260 ***	0.012		0.016	0.022	0.013	0.019	-0.226 ***
	(0.024)	(0.008)		(0.015)	(0.021)	(0.012)	(0.018)	(0.009)
Switzerland	-0.084 ***	-0.011 ***		-0.022 ***	-0.031 ***	-0.018 ***	-0.028 ***	-0.111 ***
	(0.015)	(0.004)		(0.008)	(0.011)	(0.007)	(0.010)	(0.009)
UK	0.006	-0.112 ***	0.013 ***	-0.089 ***	-0.098 **	-0.064 ***	-0.073 **	-0.161 ***
	(0.055)	(0.036)	(0.005)	(0.032)	(0.039)	(0.024)	(0.032)	(0.014)
US	-0.058	-0.104 ***	0.012 ***	-0.082 ***	-0.091 ***	-0.060 ***	-0.068 **	-0.209 ***
	(0.042)	(0.027)	(0.003)	(0.027)	(0.034)	(0.021)	(0.027)	(0.013)

Notes: The table reports the test results for safe haven and hedge properties of government bond returns against domestic stock returns. The test for safe havens is based on the estimation of Eq.(16) and the test for hedges on Eq.(17). When b_2 is not significant at the 5% level, the corresponding quadratic term is omitted from Eq.(16). For the coefficients of Eqs.(16) and (17), Newey–West heteroscedasticity and autocorrelation consistent standard errors are reported in parentheses. The total effect is calculated from $\int_l^h (b_1 + 2b_2\sigma_1)d\sigma_1 = b_1(h-l) + b_2(h^2 - l^2)$ where l denotes either the 25th or 50th percentile of stock market volatility and h denotes either the 90th or 95th percentile. The delta method is used to compute standard errors of the total effects. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.

Table 6: Estimation results for two-state regime-switching copulas: relationships between world stock returns and government bond returns

	Dep. sign	Copula	Parameter	Kendall	LL tail	UU tail	LU tail	UL tail	Tran. prob.
Australia	Positive	Gaussian	0.569 ***	0.385 ***	0	0			0.987 ***
			(0.073)	(0.056)					(0.235)
	Positive	Clayton	0.283 ***	0.124 ***	0.086	0			0.993 ***
			(0.082)	(0.032)	(0.061)				(0.118)
Canada	Positive	Gaussian	0.532 ***	0.357 ***	0	0			0.975 ***
			(0.085)	(0.064)					(0.236)
	Positive	Clayton	0.190 ***	0.087 ***	0.026	0			0.989 ***
			(0.066)	(0.028)	(0.033)				(0.117)
France	Positive	Gaussian	0.386 ***	0.252 ***	0	0			0.986 ***
			(0.056)	(0.039)					(0.106)
	Negative	R. Clayton	0.178 **	-0.082 **			0.021	0	0.984 ***
			(0.091)	(0.038)			(0.041)		(0.125)
Germany	Positive	Gaussian	0.277 ***	0.179 ***	0	0			0.986 ***
			(0.063)	(0.042)					(0.112)
	Negative	R. Gaussian	0.263 **	-0.169 **			0	0	0.973 ***
			(0.102)	(0.067)					(0.211)
Japan	Positive	Gumbel	1.029 ***	0.028	0	0.039			0.998 ***
			(0.027)	(0.026)		(0.035)			(0.100)
	Negative	R. Clayton	0.498 ***	-0.199 ***			0.248 ***	0	0.997 ***
			(0.118)	(0.038)			(0.082)		(0.150)
Switzerland	Positive	Gaussian	0.331 ***	0.215 ***	0	0			0.965 ***
			(0.072)	(0.049)					(0.134)
	Negative	R. Clayton	0.303 ***	-0.132 ***			0.102	0	0.972 ***
			(0.097)	(0.037)			(0.075)		(0.116)
UK	Positive	Gaussian	0.188 ***	0.121 ***	0	0			0.999 ***
			(0.055)	(0.036)					(0.082)
	Negative	R. Gaussian	0.392 **	-0.256 **			0	0	0.989 ***
			(0.184)	(0.127)					(0.352)
US	Negative	R. Gaussian	0.654 ***	-0.454 ***			0	0	0.951 ***
			(0.071)	(0.060)					(0.177)
	Negative	R. Clayton	0.078	-0.038			0.000	0	0.969 ***
			(0.081)	(0.038)			(0.001)		(0.115)

Notes: The table reports the results from estimating the two-state regime-switching copula model for the relationship between world stock returns and individual government bond returns. The sample period is from January 20, 1999 to June 27, 2018. "R. copula name" denotes the 90-degree clockwise rotation of the copula. "Parameter" reports the estimates of copula parameters (see Table 1). LL, UU, LU, and UL tails denote the lower–lower, upper–upper, lower–upper, and upper–lower tail dependence coefficients, respectively. Transition probabilities reported in the last column correspond to the estimates of p_{11} and p_{22}. Standard errors are reported in parentheses. The delta method is used to compute standard errors for Kendall's τ and tail dependence coefficients. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.

Table 7: Test results for safe haven (and hedge) properties of government bond returns against world stock returns

	Test for safe haven							Test for hedge
	Eq.(16)			Total effect				Eq.(17)
	b_0	b_1	b_2	25th–90th	25th–95th	50th–90th	50th–95th	b_0
Australia	0.180 ***	0.018 ***		0.031 ***	0.044 ***	0.025 ***	0.037 ***	0.217 ***
	(0.016)	(0.007)		(0.012)	(0.017)	(0.009)	(0.014)	(0.008)
Canada	0.120 ***	0.025 ***		0.044 ***	0.062 ***	0.035 ***	0.053 ***	0.172 ***
	(0.013)	(0.006)		(0.011)	(0.015)	(0.009)	(0.013)	(0.007)
France	0.207 ***	-0.081 ***	0.011 ***	-0.049 **	-0.049 *	-0.033 *	-0.033	0.098 ***
	(0.041)	(0.028)	(0.003)	(0.025)	(0.030)	(0.018)	(0.023)	(0.011)
Germany	0.175 ***	-0.083 ***	0.010 ***	-0.060 **	-0.065 **	-0.042 **	-0.047 **	0.058 ***
	(0.038)	(0.026)	(0.003)	(0.024)	(0.029)	(0.018)	(0.023)	(0.010)
Japan	-0.056 ***	-0.017 ***		-0.030 ***	-0.041 ***	-0.023 ***	-0.035 ***	-0.091 ***
	(0.016)	(0.006)		(0.011)	(0.015)	(0.008)	(0.013)	(0.009)
Switzerland	0.076 **	-0.042 *	0.007 ***	-0.016	-0.009	-0.009	-0.002	0.025 **
	(0.035)	(0.023)	(0.003)	(0.022)	(0.027)	(0.016)	(0.021)	(0.010)
UK	0.193 ***	-0.079 ***	0.009 ***	-0.068 ***	-0.079 ***	-0.049 ***	-0.061 ***	0.074 ***
	(0.033)	(0.024)	(0.003)	(0.023)	(0.028)	(0.017)	(0.022)	(0.009)
US	-0.023	-0.120 ***	0.014 ***	-0.094 ***	-0.107 ***	-0.067 ***	-0.079 ***	-0.198 ***
	(0.042)	(0.028)	(0.003)	(0.027)	(0.034)	(0.020)	(0.027)	(0.012)

Notes: The table reports the test results for safe haven and hedge properties of government bond returns against world stock returns. The test for safe havens is based on the estimation of Eq.(16) and the test for hedges on Eq.(17). When b_2 is not significant at the 5% level, the corresponding quadratic term is omitted from Eq.(16). For the coefficients of Eqs.(16) and (17), Newey–West heteroscedasticity and autocorrelation consistent standard errors are reported in parentheses. The total effect is calculated from $\int_l^h (b_1 + 2b_2\sigma_1)d\sigma_1 = b_1(h-l) + b_2(h^2 - l^2)$ where l denotes either the 25th or 50th percentile of stock market volatility and h denotes either the 90th or 95th percentile. The delta method is used to compute standard errors of the total effects. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.

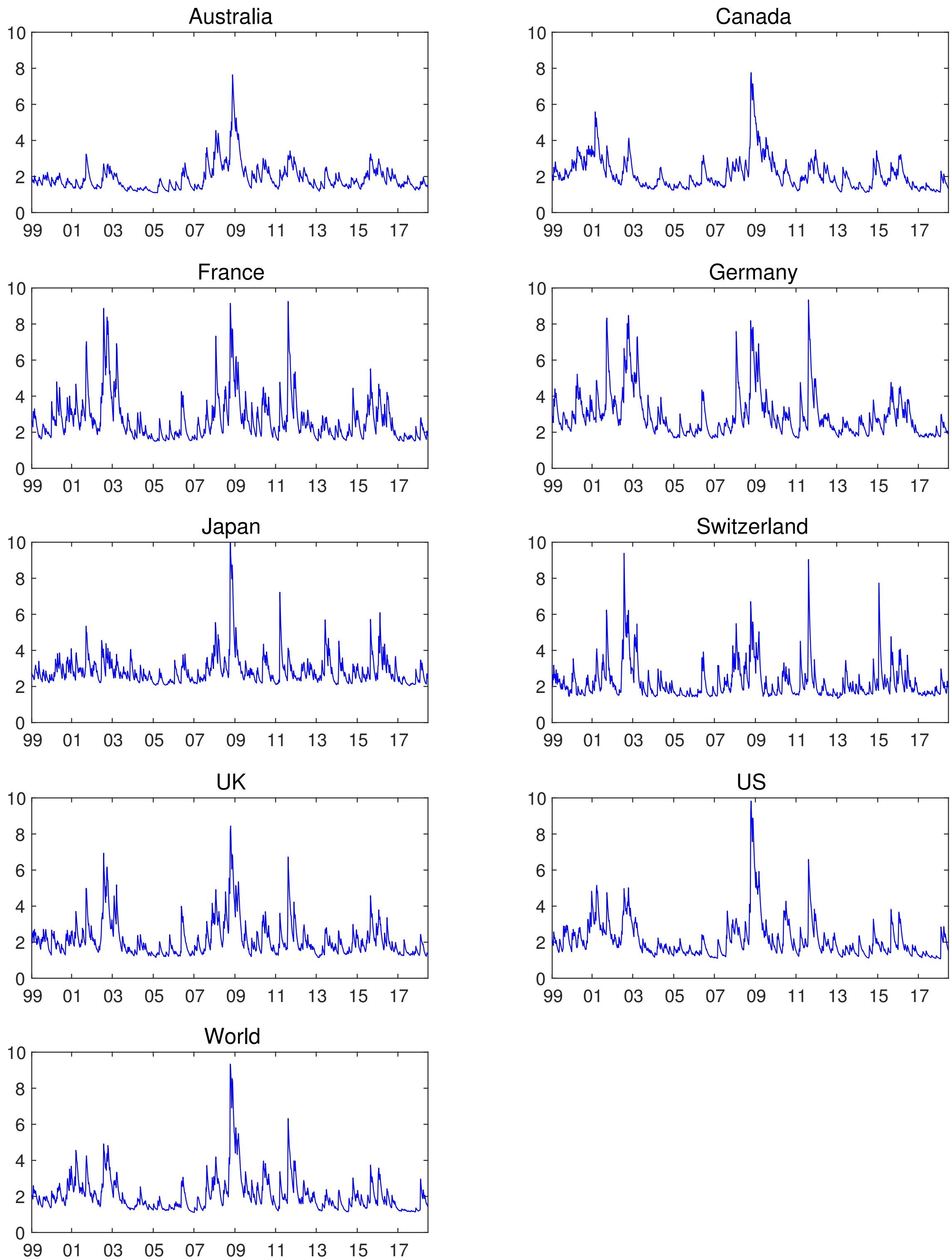

Figure 1: Volatility of stock returns

Notes: The figure shows the evolution of volatilities of individual stock returns and world stock returns, $\hat{\sigma}_{1t}$, which is calculated from the estimated GARCH models (7).

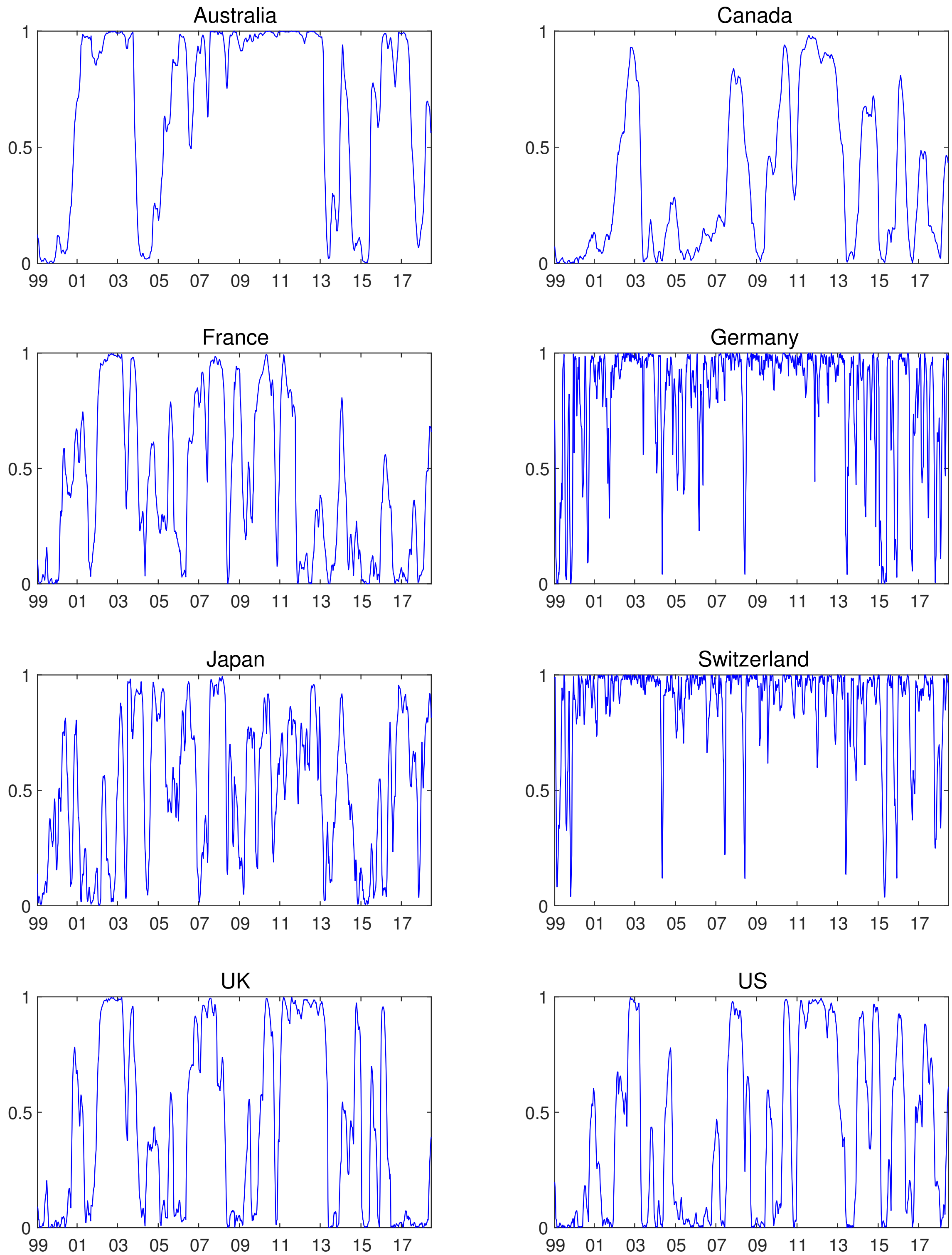

Figure 2: Smoothed probabilities for the relationships between domestic stock and government bond returns

Notes: The figure shows the evolution of the smoothed probabilities of being in a lower dependence regime, which is computed from Eq.(14), for the relationships between domestic stock and government bond returns.

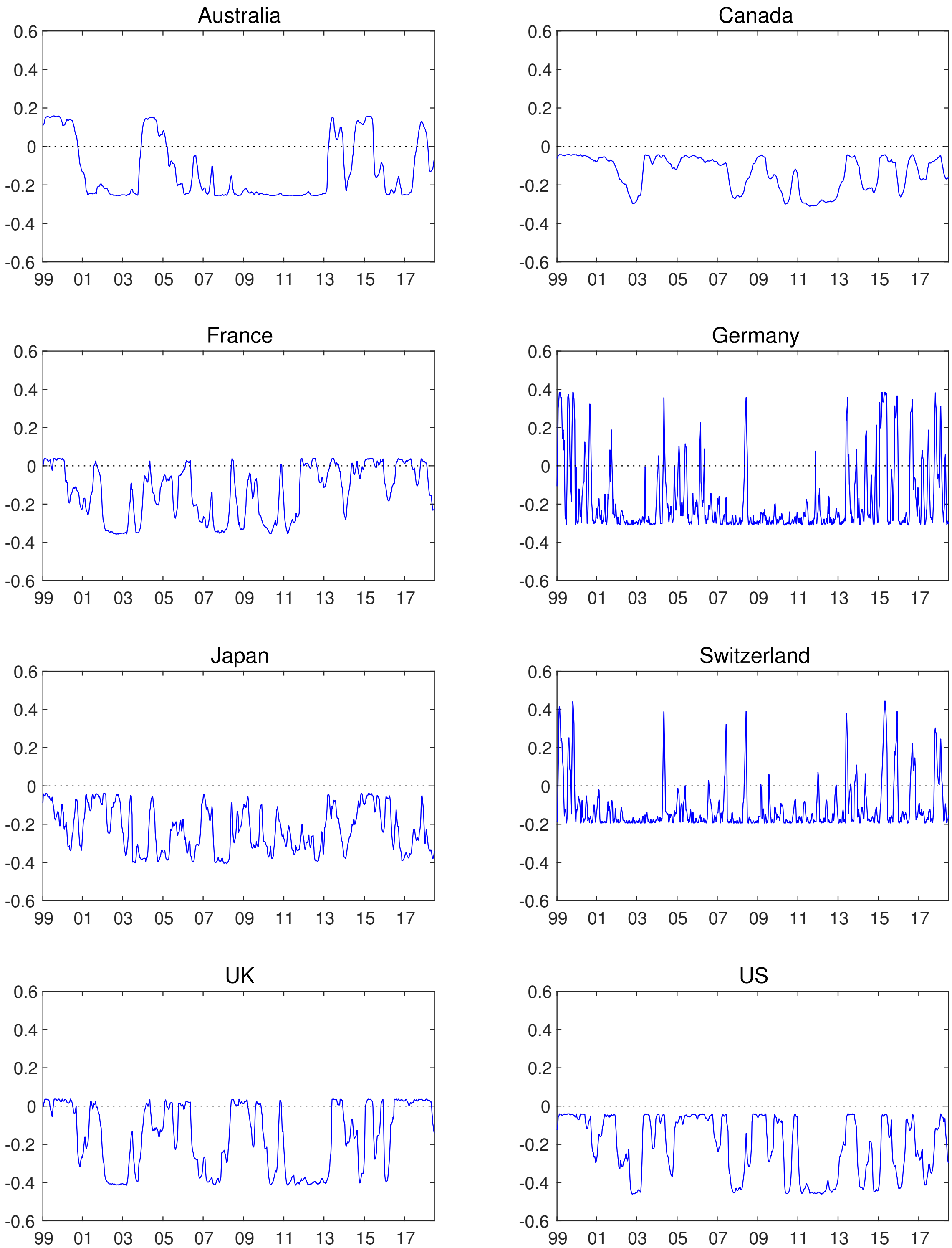

Figure 3: Smoothed Kendall's τ for the relationships between domestic stock and government bond returns

Notes: The figure shows the evolution of the smoothed Kendall's τ, which is computed from Eq.(15), for the relationships between domestic stock and government bond returns.

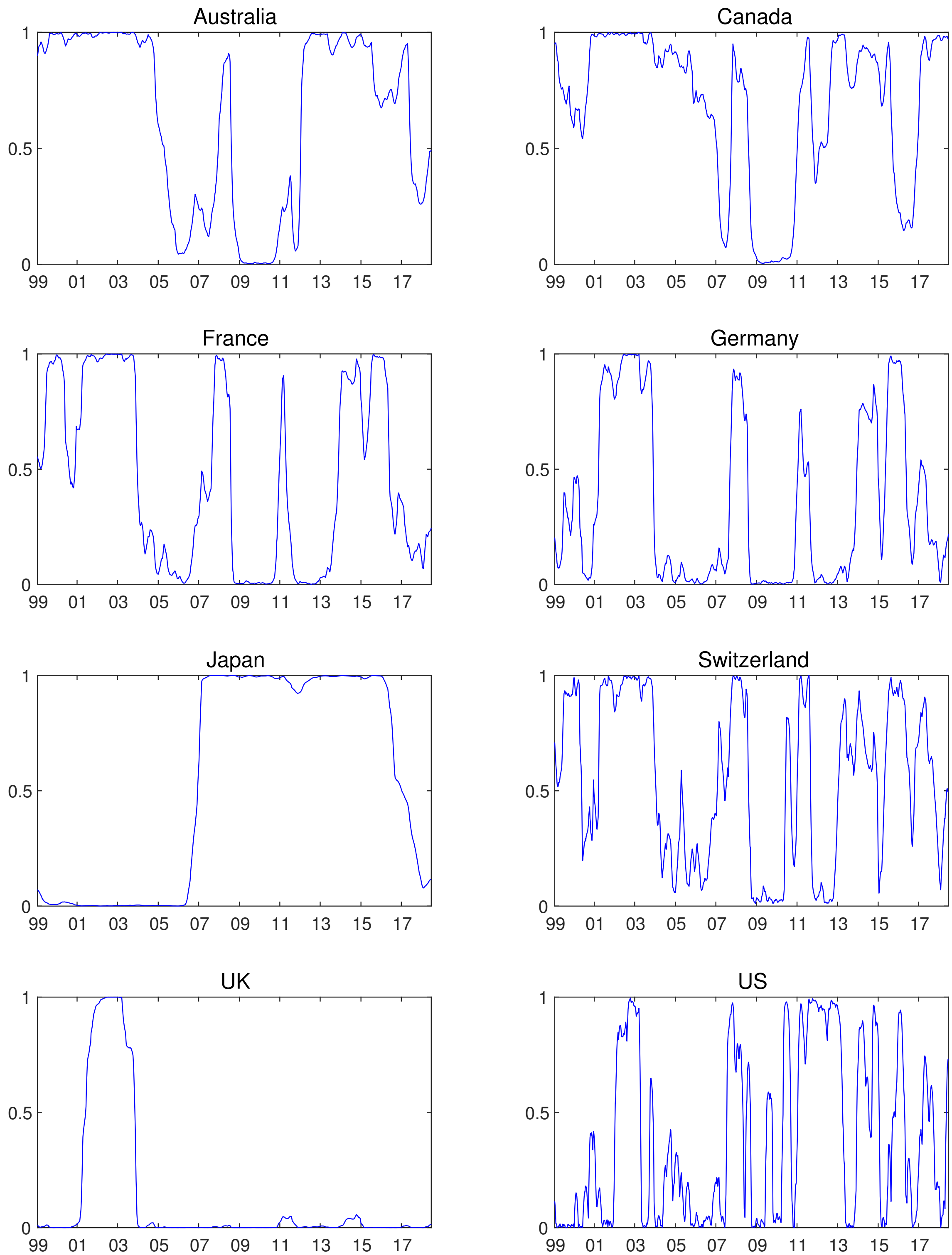

Figure 4: Smoothed probabilities for the relationships between world stock returns and government bond returns

Notes: The figure shows the evolution of the smoothed probabilities of being in a lower dependence regime, which is computed from Eq.(14), for the relationships between world stock returns and government bond returns.

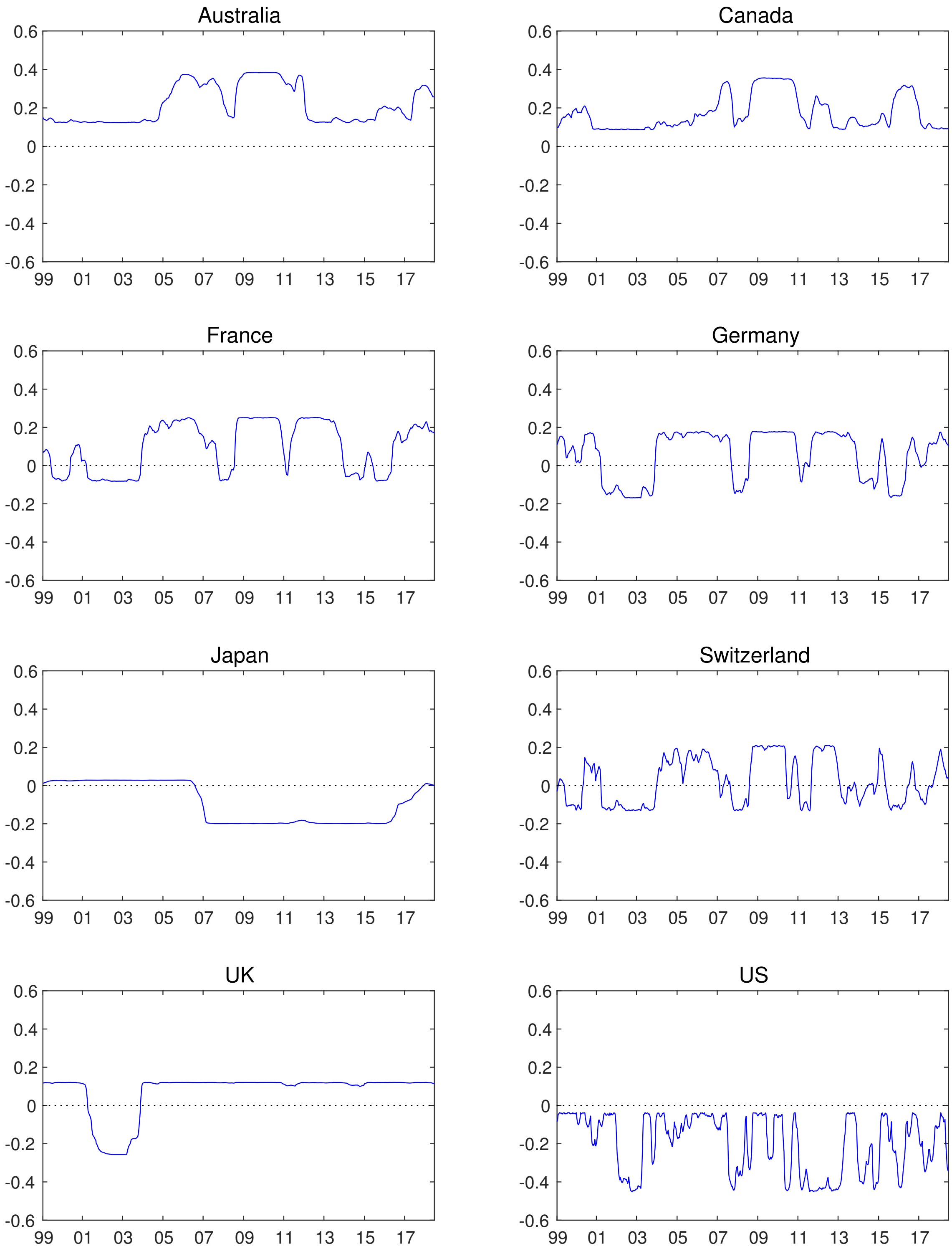

Figure 5: Smoothed Kendall's τ for the relationships between world stock returns and government bond returns

Notes: The figure shows the evolution of the smoothed Kendall's τ, which is computed from Eq.(15), for the relationships between world stock returns and government bond returns.

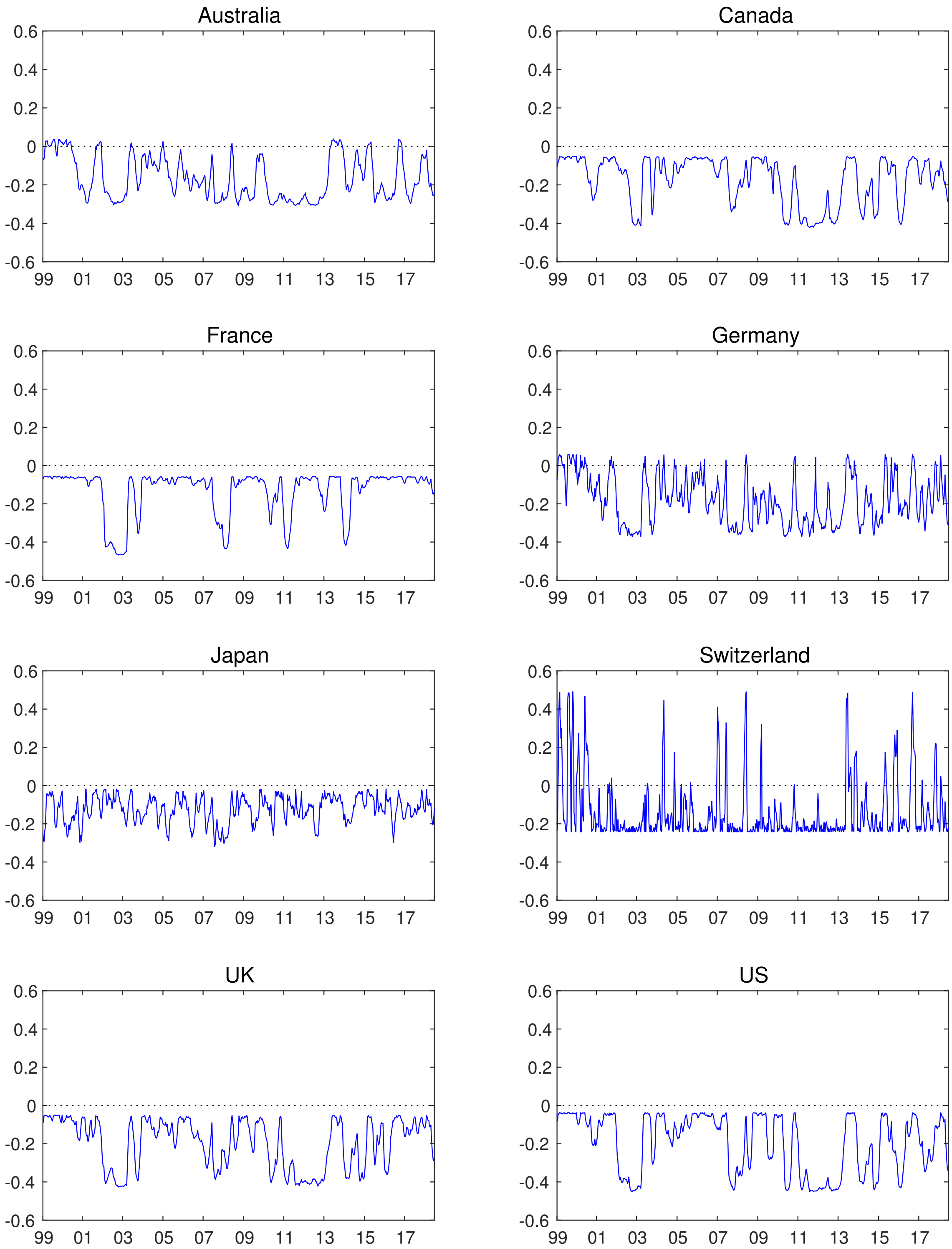

Figure 6: Smoothed Kendall's τ for the relationships between world stock returns and government bond returns in local currencies (the first sub-relations in the decomposition analysis)

Notes: The figure shows the evolution of the smoothed Kendall's τ for the first sub-relations in the decomposition analysis; that is, the relationships between world stock returns and government bond returns where the price indices used in the calculation of the government bond returns are denominated in local currencies.

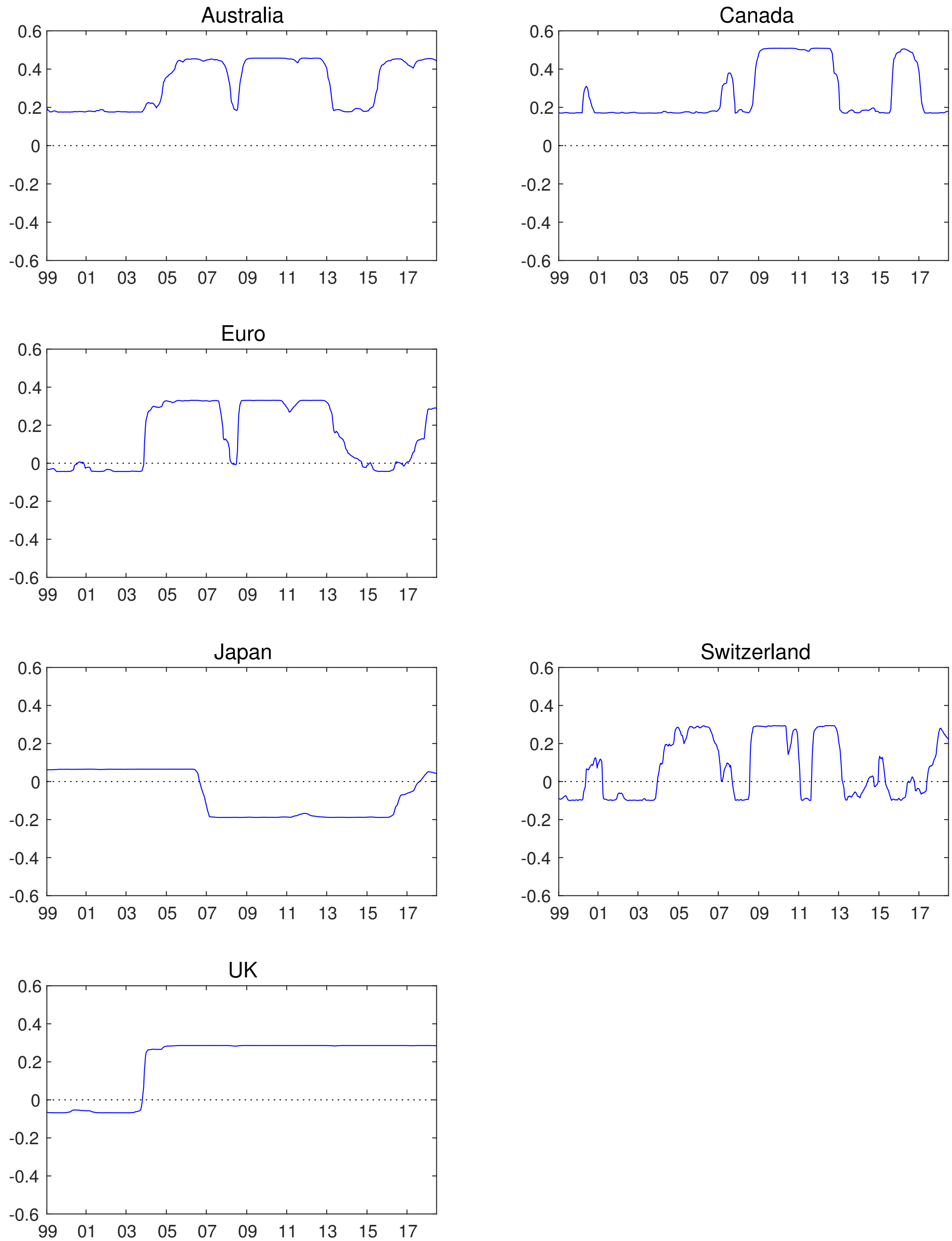

Figure 7: Smoothed Kendall's τ for the relationships between world stock returns and currency returns (the second sub-relations in the decomposition analysis)

Notes: The figure shows the evolution of the smoothed Kendall's τ for the second sub-relations in the decomposition analysis; that is, the relationships between world stock returns and currency returns where the exchange rates used in the calculation of the currency returns are expressed in terms of US dollars per unit of local currency.