A Study on Estimation of Three-Dimensional Surface Roughness and Tolerances Based on Virtual Machining Systems Considering Kinematic Motion Deviations

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A Study on Estimation of Three-Dimensional Surface Roughness and Tolerances Based on Virtual Machining Systems Considering Kinematic Motion Deviations

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Chapter 1

Introduction
1. Introduction

1.1 Preface

In the most recent decade, revolutions in the machine industries are constantly exploring systems and methods to increase the quality and the reliability of the products and to decrease the costs of the machining operations. Nowadays, the proposed machining processes contain collaboration with advances in digital engineering technologies. CAD/CAE/CAM systems are now being widely applied to design, analysis and manufacturing processes of mechanical products. Virtual manufacturing simulation is a digital engineering technology which can prove an important tool to reduce the developmental costs and the time required for physical prototyping in order to estimate machining accuracy, surface roughness and the geometric dimensioning and tolerancing which is one of the most important characteristics of the Computer Numerical Control (CNC) machine tools for generating the products with high accuracy and complicated geometries.

The research summarized in this doctoral dissertation deals with a simulation based estimation of 3-dimensional (3D) surface roughness and tolerances including kinematic motion deviations. This dissertation presents the basic concepts and study of virtual machining of boring and turning processes, the estimation of the 3D surface roughness and the 3D tolerances, based on geometric tolerances and the kinematic motion deviations of the CNC machining centers and CNC turning centers.

1.2 Background

The accuracy of a machine tool affects the quality of finished products and also the ability for economic production. Nowadays, the needs for precision machining are mainly driven by IC-technology, liquid crystal display, and plasma display. Since the integrated circuit was invented, in 1958, the need to increase the number of transistors on a single chip has been growing higher and higher. Many activities in the industry and academia have been aimed at increasing the precision of positioning and machining to create high precision products and processes. Manufacturing for precise products concerns high accuracy in size, shape and surface quality. To achieve a high accuracy, certain factors have to be understood from the design to the manufacturing process, such as interaction between tools and workpieces, behaviors of machine tools under thermal, dynamic and kinematic loading. The quality of the finished products is largely influenced by the following factors:

- Deviations from the planned relative movement between the tools or tool carriers and the workpieces or workpiece supports.
- Wear conditions and elastic deformations of the tools.
- Elastic deformations of the workpieces and clamping elements.
The deviations from the defined relative motions between the tools and workpieces may be grouped into geometric and kinematic categories. The geometric deviation consists of positional inaccuracies and errors in the shape of machine components (tables, tool holders, guides, etc.). The kinematic deviation occurs in coordinate movements, i.e. function movements (thread cutting, guide-way control etc.). These errors are transmitted into the accuracy of the workpieces in varying amounts, depending on the particular production conditions, which apply in the given case. Both types of deviations are the results of production and assembly errors in the elements which are used in the machine construction (Nerdnoi, 1999).

Various types of CNC machine tools are now being designed and applied to machining processes of complicated machine products. The machining accuracy is one of the most important characteristics of the CNC machine tools for generating products with high accuracy and complicated geometries. Some researchs have been carried out to analyze the machining accuracy of the machine tools based on the deviations of the shape generation motions between the tools and the workpieces (Sugimura, et al., 1981; Reshetov, et al., 1988; Sakamoto, et al., 1994; Sugimura, et al., 1998). However, the kinematic motion deviations of the machine tools are strongly influenced by the geometric deviations of the components, such as guide ways and bearings. Therefore, it is now required to clarify the relationships between the kinematic motion deviations of the machine tools and the geometric deviations of the components, from the viewpoints of the design and the manufacturing of the machine tools and their components.

With regard to the geometric deviations of the machine components, much research work has been carried out to deal with the dimensional tolerances and the geometric tolerances, aimed at realizing systematic analysis and design methodologies for the three dimensional machine products (Roy, et al., 1991; Voelcker, 1993; Nigam and Tuner, 1995; Ngoi, et al., 2000). However, the relationships between the kinematic motion deviations and the geometric deviations of the components have not yet been clarified, as the linear and rotary tables of the machine tools are supported by more than two-guide ways and that all the geometric deviations of individual guide ways affect the kinematic motion deviations of the linear and rotary tables.

Machining processes on the CNC machine tools for generating products with high accuracy and complicated geometries are inherently complex, and lead to use empirical methods for process developments. In particular, process parameters such as machining speeds, feed rates and tooling are usually selected based on handbooks and trial-and-error prototyping. However, these methods do not guarantee the process parameters which satisfy the required quality (Ramaswami, 2010).

Since the early 1990s, a paradigm shift in manufacturing from ‘real’ to ‘virtual’ production has resulted in a build-up of research interests in the simulation techniques. With the aid of computers, it becomes possible to simulate some of the activities of physical manufacturing systems. The main objective of the virtual productions is to understand and to emulate the
behavior of the manufacturing systems on the computers prior to the physical productions, aiming at reducing the amount of testing and experiments on the shop floors. They are therefore called virtual manufacturing, virtual machine tools, virtual machining, virtual assembly, virtual tooling and virtual prototyping (Abdul Kadir, et al., 2011). Some simulation models for the machining processes have recently been proposed to predict the dimensions, geometries and geometric deviations of the machined parts, aiming at investigating the suitable machining process parameters (Govik, et al., 2012; Luo, et al., 2010). However, the proposed models have been applied only to the limited areas of the machining processes and have not yet considered the verification of the generated faces of boring and turning processes including kinematic motion deviations.

Once the parts are machined on CNC machine tools, the surface roughness and geometric dimensioning and tolerancing need to be inspected to ensure compliance with the design specifications. With increasing focus on precision manufacturing industries, especially in the automotive, electronic, defense and aerospace sectors, 100% inspection of parts is mandated to ensure product reliability and functioning (Ramaswami, 2010). Therefore, the proposed model for the estimation of the surface roughness and the geometric dimensioning and tolerancing have been applied to the virtual machining simulations which can estimate and improve the machining processes before real machined parts. The simulation systems have become an important tool to reduce the development cost and time required for physical prototyping, the product design and the process planning for select the machining process parameters. It can also reduce the lead time to market, and improve the responsiveness and competitiveness of the manufacturers. It is essential to establish effective and efficient estimation methods of the surface roughness and the geometric dimensions and tolerances for the industrial products such as automotive, electronics and aerospace parts, in order to meet the keen competition of the global economy.

1.3 Objectives of the Research

Surface roughness and tolerances are important in product design and manufacturing, which deal with checking the feasibility and quality of individual parts and assembly parts, because it affects not only the performance of the products, but also the costs. To facilitate the prediction and adaptation of machined parts in CNC machine tools, a new appropriate methodology is needed to produce and to estimate the surface roughness and tolerances before the real and physical machining processes. This will facilitate the adaptation to changes in the machine industries, in order to increase the quality and reliability of their products and reduce the costs of the machining processes.

The virtual machining approaches are becoming increasingly important for estimation of 3D surface roughness and tolerances of machining processes including kinematic motion deviations. The virtual machining provides a systematic way to estimate the 3D surface roughness and tolerances with the accuracy and the reliability of the products and manufacturing processes.
This research applies a virtual machining model to estimation of the 3D surface roughness and tolerances including kinematic motion deviations of boring and turning processes. The objective of this research is to develop a virtual machining model for bored and turned parts to analyze the effect of machining process parameters on the quality of the final products. A virtual part profile can be generated using the virtual machining system, based on the design specifications, machining process parameters, including the kinematic motion deviations of the machine tools. The part profiles are thus created to investigate the effects of the machining parameters along with the static errors in the machine beds, spindle errors and tool geometries, and to study the capability of the machine tools to produce parts meeting the design requirements. The objectives of this research are listed as follows;

**Simulation of the machining processes including kinematic motion deviations**

The objective of this section research is to propose a simulation model of the machine tools and the machining processes in which a simulation model is proposed and applied to the estimation of virtual machining in boring and turning processes including kinematic motion deviations. A set of points on the bored and turned faces are obtained through the simulation. The proposed model represents the boring and turning processes based on both the shape generation motions and the cutting tool geometries. The individual motions are mathematically described by 4 by 4 transformation matrices including the kinematic motion deviations. Emphasis is given to the modeling and analysis of the boring and turning processes of the single point tools.

**Estimation of the 3D surface roughness including kinematic motion deviations**

The objective of this section research is to propose an estimation of 3D surface roughness based on simulation of virtual machining in boring and turning processes including kinematic motion deviations on CNC machining centers and CNC turning centers, respectively. A simulation model is proposed to represent the boring process with kinematic motion deviations of the spindles of the milling machines, and to represent the turning process with kinematic motion deviations of the spindles, carriages and cross slides of the turning machines. A set of points on the bored and turned faces are obtained through the simulation. The proposed model represents the boring and turning processes based on both the shape generation motions and the cutting tool geometries. The individual motions are mathematically described by combining 4 by 4 transformation matrices including the kinematic motion deviations. Emphasis is given to the modeling and analysis of the boring and turning process of the single point tools. A systematic method is also proposed to verify the 2D and 3D surface roughness of the bored and turned faces base on the simulation results.

**Analysis of the kinematic motion deviations of machining centers and estimation of the 3D tolerances including kinematic motion deviations**
The objective of this section research is to establish mathematical models representing the kinematic motion deviations of the machining centers, on the basis of the geometric tolerances of the components, and to apply the models to theoretical analysis of both the kinematic motion deviations of the machining centers and the estimation of 3D tolerances. In the previous papers (Satonaka, et al., 2007b; Satonaka, et al., 2008), a mathematical model has been proposed to represent the kinematic motion deviations of the linear tables and the proposed model was applied to the analysis of the kinematic motion deviations of the linear tables and the 3-axis machining centers. However, the rotary tables have not yet been discussed in detail (Watabiki, et al., 2009). The kinematic motion deviations of the machining centers are estimated based on the models in the various conditions of the geometric tolerances of the components and the various table positions, to investigate the influence of the geometric tolerances of the components on the kinematic motion deviations of the five-axis machining centers.

The developed model of the machine tools are applied to the estimation of 3D tolerances based on the kinematic motion deviations on CNC machining centers and CNC turning centers. The simulation of virtual machining model is expanded to estimate the 3D tolerances both the boring and turning processes with kinematic motion deviations. A set of points on the bored and turned faces are obtained through the simulation. The proposed model represents the boring and turning processes based on both the shape generation motions and the cutting tool geometries. The individual motions are mathematically described by combining 4 by 4 transformation matrices including the kinematic motion deviations. Emphasis is given to the modeling and analysis of the boring and turning processes of the single point tools. A systematic method is also proposed to verify the 3D tolerances of the bored and turned faces based on the simulation results.

1.4 Organization of the Dissertation

The organization of the dissertation chapters and their relationships are shown in the Fig. 1-1. Chapter 1 provides a brief introduction of the historical background of the research, and clarifies the necessity and objective of the research.

Chapter 2 reviews the background and importance of the conventional methods and new approaches for analysis kinematic motion deviations and the geometric deviations, virtual machining, 3D surface roughness and 3D tolerances based on the literature survey, new areas for research are identified and proposed.

Chapter 3 discusses the virtual machining model representing the virtual machining centers for the boring processes and virtual turning centers for the turning processes, which include the kinematic motion deviations. The bored and turned faces generated are obtained as a set of points and evaluated to investigate the effect of various process parameters on the geometry of the parts. Once the part is verified virtually, the performance of the machining process parameters to
generate a part that meets the required quality specifications can be investigated for estimation
the 3D surface roughness and tolerances based on the simulation results.

Chapter 1
Introduction

Chapter 2
Review of Relevant Literature

Chapter 3
Simulation of virtual machining systems in boring and turning processes

Chapter 4
Estimation of 3-dimensional surface roughness including kinematic motion deviations

Chapter 5
Estimation of 3-dimensional tolerances including kinematic motion deviations

Chapter 6
Conclusions

Fig. 1-1 Dissertation chapters and their relationship

Chapter 4 discusses the estimation of 3D surface roughness of produced by virtual boring and
turning machining processes. The estimation of 3D surface roughness estimated both the 2-
dimensional (2D) and 3D through the boring and turning process simulations with kinematic
motion deviations. A model is proposed to represent the kinematic motions of the cutting edges
against the workpieces, taking into consideration the kinematic deviations of the machining
centers and the turning centers. Details of the virtual machining module are proposed to estimate
the geometric deviations of the machined face based on the kinematic motions of the cutting
edges. A proposed model is applied to the simulation of the simple boring and turning process,
the geometries of the machined faces are estimated, based on the cutting conditions, the tool geometries and the kinematic deviations of the boring and turning processes. A method is also proposed to estimate both the 2D and 3D surface roughness based on the boring and turning process simulation with the kinematic motion deviations. Case studies are shown with examples of the generated faces by boring and turning process simulations considering kinematic deviations in order to estimate the 3D surface roughness.

Chapter 5 discusses the estimation of 3D tolerances including kinematic motion deviations. In the beginning, a mathematical model of kinematic motion deviations of machine tools is discussed on the basis of the geometric tolerances. The shape generation motions are bases for analysis of machine tools including both the linear tables and rotary tables. The 3D tolerances of boring and turning processes are analysed based on the machine tool models including kinematic motion deviations. A systematic method is proposed in this section to simulate the shape generation processes in both the boring and turning operations, to estimate the geometric dimensioning and tolerancing of both bored and turned faces, based on the machining parameters. The shape generation motions with deviations are mathematically described by combining 4 by 4 transformation matrices. A set of points on the bored and turned faces are generated through the simulations, and an assessment surface is obtained as the datum reference to estimate the 3D tolerances, based on the points generated by the boring and turning process simulations.

Chapter 6 summarizes the doctoral dissertation.
Chapter 2

Review of Relevant Literatures
2. Review of Relevant Literatures

The literatures related with simulation based estimation of 3-dimensional surface roughness and tolerances including kinematic motion deviations are reviewed briefly in this chapter and discussed below.

2.1 Kinematic motion deviations

Various types of CNC machine tools are now being designed and used in machining processes generating complicated machine products. The machining accuracy is one of the most important characteristics of these machine tools because the products may have highly complicated geometries.

Accuracy can be defined as the degree of agreement or conformance of a finished part with the required dimensional and geometrical accuracy (Jedrzejewski and Modrzycki, 1997; Ramesh, et al., 2000). Error, on the other hand, can be understood as any deviation in the position of the tool’s cutting edge from the position theoretically required to produce a workpiece of the specified tolerance (Morris, 1997; Ramesh, et al., 2000). The errors can be classified into two categories namely quasi-static errors and dynamic errors;

Quasi-static errors are deviations between the tool and the workpiece that are slowly varying with time and related to the structure of the machine tool itself. These sources include the geometric/kinematic errors, errors due to dead weight of the machine’s components and errors due to thermally induced strains in the machine tool structure.

Dynamic errors, on the other hand, are caused by sources such as spindle error motion, vibrations of the machine structure, controller errors etc. These are more dependent on the particular operating conditions of the machine.

The geometric/kinematic deviations are quasi-static error sources between the tool and workpiece of machine tool. In general machine tools, especially machining center and turning center consist of a bed, column, spindle and its slide and the various linear and/or rotary axes. Each of these elements contributes to the total deviations of the system that is represented by the error budget. Therefore, errors can broadly be grouped into three major classes namely geometric and kinematic errors, temperature induced errors or thermal errors and cutting forces induced errors. An overview of the error budget of a machine tool and the factors affecting the same is given in Fig. 2-1. While these are the major contributors to the volumetric error of a machine tool, other errors also contribute to the overall error, though not as significantly (Ramesh, et al., 2000).
The geometric/kinematic deviation shows as above is the most problem in the error budget in a machine tool which is quasi-static error sources between the tool and workpiece of machine tool, account for about 70 percent of the total error of the machine tool which are a major focus of error compensation research (Evans, 1996; Ramesh, et al., 2000). Therefore, in this research focus on the literature review of geometric and kinematic errors.

Geometric deviations are errors that are extant in a machine on account of its basic design, inaccuracies built-in during assembly and as a result of the components used in the machine. As such, they form one of the biggest sources of inaccuracy. These errors are concerned with the quasi-static accuracy of surfaces moving relative to one another, like surface straightness, surface roughness, bearing pre-loads etc. Geometric errors have various components like linear displacement error (positioning accuracy), straightness and flatness of movement of the axis, spindle inclination angle, squareness error, backlash error etc (Ni, 1997; Ramesh, et al., 2000).

Kinematic deviations are concerned with the relative motion errors of several moving machine components that need to move in accordance with precise functional requirements. These errors are particularly significant during the combined motion of different axes as in the case of gear hobbing or profile machining where co-ordination of rotary with respect to linear axes or linear with respect to linear axes is of prime importance. Such errors occur during the execution of linear, circular or other types of interpolation algorithms and are more pronounced during actual machining (Ramesh, et al., 2000).
The modeling of geometric/kinematic errors in machine tools has been reported for a couple of decades. Some research work has been carried out for the investigation of the geometric/kinematic deviations modeling of machine tool (Sugimura, et al., 1981; Kiridena and Ferreira, 1994; Portman and Inasaki, 1996; Sugimura and Murabe, 1997; Okafor and Ertekin, 2000; Ramesh, et al., 2000a, 2000b; Bohez, 2002; Hsu and Wang, 2007; Satonaka, et al., 2008; Uddin, et al., 2009; Ibaraki, et al., 2012). The work can be divided into three categories; the first set of papers mainly focuses on developing a model for geometric/kinematic errors and other errors such as thermal, vibration, and cutting forces related errors etc. The second set includes error compensation algorithms also and describes various means to compensate for errors. The third set mainly focuses on developing of the kinematic deviations model of the machine tools based on the shape generation motions between the tools and the workpieces in order to simulate and estimate in the machining accuracy. Keeping in line with the objectives of my current research, the literature review will focus on aspects of the kinematic deviations model based on the shape generation motions between the tools and the workpieces. Some of the key papers in the area of kinematic motion deviations based on the shape generation motions modeling are discussed below.

One of the earliest discussions of the modeling of the shape generation motions of machine tool can be found in Sugimura et al. (1981). The approach to the modeling taken by Sugimura et al. (1981) is similar to the approach of Reshetov and Portman (1988) and Moriwaki et al. (1993). Inasaki (1994) has proposed a theory of generation motions for machine tools, defined the mathematical model of a machine tool and the coordinate transformation matrices. The shape generation motion can be expressed by a chain of relative motions. Portman and Inasaki (1996) have proposed a theory of form-shaping systems and its application to grinding machines which is similar to approaches with shape generation motions. The form-shaping system enables the simulation of the form-shaping process, and takes into account machine tool error, facilitating quantitative investigation of the influence of this error on the machining accuracy. The computer simulations and the actual grinding tests reveal the practicality of the system for analysis of the machining accuracy.

Sugimura and Murabe (1997) have proposed a study of analysis of alignment errors of 5-axis machine tools. The research has developed a systematic design method of multi-axis machine tools based on a mathematical model representing the shape generation processes of the machine tools. The model describes the relative motions between the tool and the workpiece by combining 4 by 4 homogeneous transformation matrices. Machining errors of various types of 5-axis machine tools are investigated through simulation by applying the proposed model, aimed at verifying adaptability of the feed axis configurations of the machine tools from the viewpoint of the machining errors.

Tsutsumi and Saito (2004) have proposed the identification of angular positional deviations inherent to 5-axis machining centers with a tilting-rotary table by simultaneous four-axis control
movements. The paper aims to propose a method of estimating the eight deviations within the thirteen systematic deviations in a five-axis machining center which Inasaki et al. (1997) has pointed out in the shape generation theory of machine tools - Its basis and applications. The validity of the proposed method has been confirmed by simulation. Four of the eight deviations are identified by applying both the measured data by a ball bar and the reference data calculated for the simultaneous four-axis control movement to the observation equation. The other four deviations are geometrically identified. The proposed method is effective for identifying the deviations inherent to five-axis machining centers.

Uddin et al. (2009) have proposed the prediction and compensation of machining geometric errors of five-axis machining centers with kinematic errors. This paper presents a scheme to predict and evaluate the machining accuracy of a five-axis machining center with its kinematic errors. Three kinematic errors associated with linear axes and eight kinematic errors associated with rotary axes of the machining center which carried out in the shape generation theory of machine tools by Inasaki et al. (1997) are considered and identified practically by a double ball bar (DBB) method. By using an error model with the kinematic errors, three-dimensional interference of the tool and the workpiece is calculated along the tool path to simulate the machining geometric error under the influence of the machine’s kinematic errors.

Ibaraki et al. (2010) have proposed machining tests for a five-axis machine tool such that its kinematic errors can be separately identified by evaluating the geometric error of finished workpieces. In each machining pattern, a simple straight side cutting using a straight end mill is performed. Experimental results demonstrate that the influence of kinematic errors can be observed in a very comprehensive manner on error profiles of finished workpieces. The machining tests presented in this paper is to identify eight kinematic errors associated with rotary axes defined in the shape generation theory of machine tools Inasaki et al. (1997) have pointed out.

Although extensive research has been conducted to model the effect of kinematic motion deviations and analyze the machining accuracy of the machine tools based on the deviations of the shape generation motions between the tools and the workpieces. However, the kinematic motion deviations of the machine tools are deeply influenced by the geometric deviations of the components, such as guide ways and bearings. Therefore, it is now required to clarify the relationships between the kinematic motion deviations of the machine tools and the geometric deviations of the components, from the viewpoints of the design and the manufacturing of the machine tools and their components.

In this research, the shape generation motions method has been used to model the kinematic motion deviations in the machine tool, on the basis of the geometric tolerances of the components and virtual machining in order to estimate the three-dimensional (3D) surface roughness and tolerances.
2.2 Virtual Machining

Since the early 1990s, a paradigm shift in manufacturing from ‘real’ to ‘virtual’ production has resulted in a build-up of research interests in the simulation techniques. With the aid of computers, it becomes possible to simulate some of the activities of physical manufacturing systems. The main objective of the virtual productions is to understand and to emulate the behavior of the manufacturing systems on the computers prior to the physical productions, aiming at reducing the amount of testing and experiments on the shop floors. They are so called virtual manufacturing, virtual machine tools, virtual machining, virtual assembly, virtual tooling and virtual prototyping (Abdul Kadir, et al., 2011).

![Fig. 2-2 Architecture of a virtual machining system (Altintas, et al., 2014)](image)

The virtual machining of parts by considering the physics of the manufacturing processes has recently been evolving. The virtual machining concept is illustrated in Fig. 2-2. The Computer Aided Design (CAD) model of the part is used to generate Numerical Control (NC) programs in a Computer Aided Manufacturing (CAM) environment where the process planners design tool path strategies and select cutting conditions based on their experience. The NC program is tried on a physical machine, and if the process is found to be faulty, the trial and error cycle between the CAM and physical machining steps is repeated until a satisfactory result is obtained. The aim of the virtual machining is to reduce or even eliminate physical trials by simulating the physical operations in digital environments ahead of costly production as introduced by Altintas in 1991 (Altintas and Spence, 1991; Altintas, et al., 2014).

Earlier research in this area of virtual machining is mostly focused on machining processes such as: milling processes, turning processes, grinding processes etc. The analysis is involved in the process optimization, numerical analysis and parameter prediction such as force estimation, material removal rate, machining error, machining time, feed, temperature etc. Keeping in line with objectives of the present research, the literature review will focus on aspects of the virtual machining on analysis of boring and turning processes. Therefore, several researchers have focused on development of models for various virtual machining and analysis of boring and turning processes are discussed below.
2.2.1 Boring process modeling

Considerable work has been reported on virtual machining modeling of boring processes. The motivation for most of this research has been improved numerical analysis and parameter prediction such as cutting force estimations, machining error, feed, temperature etc. Rigal et al. (1998) have proposed a model for simulation of vibrations during boring operations of complex surfaces. The proposed procedure integrates a finite element method (FEM) analysis and a linear model of the cutting process based on equivalent cutting edge geometry. On the basis of a parameterized model of the tool-material interaction, a linear FEM model describing the behaviour of the workpiece, tool and their contact is presented. The frequency responses are processed and the simulation extends with a mixed procedure (FEM analysis and simulation using Matlab). However, this research did not consider the machining process analysis and have not yet considered the verification of the generated faces.

Recently, some models have been proposed for both the machine tools and the machining processes and applied to the analysis of such characteristics as geometric deviations, kinematic deviations, and dynamics of the machine tools, the machining processes and thermal expansion in machine tools.

Lazoglu et al. (2002) have proposed dynamics of boring processes: Part III-time domain modeling. This article presents a mathematical model and a computational algorithm for time domain model of boring process dynamics. The model was developed in a modular structure. It consists of a workpiece geometry and surface topography module, kinematics and tool position module, dynamic chip load module, dynamic cutting force prediction module and structural dynamics module. The time domain model takes the cutting process parameters, tool and workpiece geometries and the modal parameters as inputs. It predicts instantaneous cutting forces and vibrations along the machining time, and machined workpiece topography as outputs. Some of the simulated and experimental results for various cutting conditions were presented and compared for validation purposes. The experimental and simulation results show good agreement.

Atabey et al. (2003a, 2003b) have proposed mechanics of boring operations. The mechanics model presented in this paper is used in predicting the cutting forces generated by inserted boring heads with runouts and presented in multi-insert boring heads. This article presents a mathematical model for the cutting force systems as a function of tool geometry, chip load, cutting edge contact length and process parameters (such as feed rate, cutting speed, radial depth of cut) based on the physics, kinematics and mechanics of the boring process. The model developed here can be used in the process planning of boring operations with inserted boring heads so that the surface finish and dimensional quality of the holes are maintained by avoiding excessive forced vibrations.
However, the entire proposed models have been applied only to the limited areas of the machining processes and have not yet considered the verification of the generated faces.

Tang and Sasahara (2007), (2008a) and Tang et al. (2008b) have proposed an investigation of thermal behavior and machining error resulting from the cutting forces and thermal expansion on cylinder liner during its boring process. The research was conducted on the effect of cutting conditions on temperature rise of cylinder liner during cylinder-boring process. The influence of the cutting conditions (for example: cutting speed, feed rate, depth of cut, cutting fluid) on the cutting forces and temperature distribution in a cylinder during machining was investigated, and the heat partition flowing to the workpiece under various cutting conditions (especially, under high-speed cutting, with a maximum cutting speed up to 900 m/min) was then determined. A three-dimensional FEM analysis model was developed to simulate the temperature distribution in the cylinder liner and thermal expansion of the cylinder liner during machining. Then, the suitable cutting conditions in cylinder boring were confirmed by FEM analysis. However, the entire proposed model did not deal with how the individual models are integrated to estimate the final geometries of the machined parts by considering the inserted tools in accordance with the international tool geometry standards.

Kaymakci et al. (2012) have proposed a unified cutting force model for turning, boring, drilling and milling operations with the inserted tools. The inserted tool tips and their orientations to the reference tool coordinates are mathematically represented by International Standardization Organization (ISO) tool definition standards. The friction forces acting on the rake faces are transformed into reference tool coordinates using the general transformation matrix. The material and insert geometry-dependent friction and normal forces are transformed into a common reference frame followed by the operation specific machine coordinates. It is shown that one unified model is capable of predicting forces for multiple metal cutting operations. The generalized modeling of metal cutting operations allows the simulation of part machining with multiple operations and various tools. The model will be extended to develop unified chatter stability laws for multiple operations. However, the proposed model did not deal with how the individual models are integrated to estimate the final geometries of the bored faces, the geometric deviations and the surface roughness in the virtual machining processes.

Although extensive research has been conducted to develop several models in virtual machining, focusing on development of models for boring processes, However, the virtual machining of parts by considering the physics of the boring processes including kinematic motion deviations of machine tools as relative motion between the tool and workpiece, which are deeply influenced by the geometric deviations of the products, has not yet been clarified. Therefore, it is now required to clarify the relationships between the kinematic motion deviations of the machine tools and the geometric deviations of models of the boring processes. In the present research, the virtual machining in boring processes is proposed to simulate model of the machine tools and the machining processes in order to estimate and to verify the geometric deviations of the bored faces.
on CNC machining centers. The proposed here represents the boring processes based on both the shape generation motions by considering the machine tool coordinate systems according to ISO 230-1 latest issues and the cutting tool geometries in accordance with the international tool geometry standards.

2.2.2 Turning process modeling

Considerable work has been reported on virtual machining modeling of turning processes. The motivation for most of this research has been improved similarity with the boring processes in numerical analysis and parameter prediction such as cutting forces estimation, machining error, feed, temperature etc.

Hong and Ehmann (1995) have proposed a method for generating engineered surfaces by the surface-shaping system. Their method defined a generalized analytical model and procedure for the simulation of the surface generation process. The basic idea of the surface generation process is introduced first. Then, the cutting tool geometry is defined. General concepts of cutting tool geometry are applied to the majority of metal removal based surface generating processes. Specifically, the major cutting edge region is defined by a surface-surface intersection algorithm and the cutting angles are obtained along the major cutting edges. Thereafter, by improving the existing method for modeling machine tool kinematics, embodied in the model of the "form-shaping system" proposed by Reshetov and Portman (1988). The paper also presented an application of the method of the surface-shaping system of in turning processes. However, the paper does not discuss how the individual models are combined to obtain the final geometry of the turned faces including inserted tool tips, and their orientations to the reference tool coordinates are mathematically represented by ISO tool definition standards.

In later research of the virtual machining in turning processes, several researches have proposed the simulation of cutting force models. Rao and Shin (1999) have proposed dynamic cutting force processes for the three-dimensional or oblique turning operation chatter prediction in turning. The dynamic force model was implemented on a computer to generate time-saving chatter predictions. Wang (2001) has proposed a development in predicting the chip flow direction and cutting force models for turning operations. The model is presented which predicts the chip flow direction in turning operations with nose radius tools under oblique cutting conditions. The model presented only the tool cutting edge geometry and the cutting conditions (feed and depth of cut). The paper also compared the results of experiments which have verified the chip flow model and shown that the model’s predictions are in good agreement with the experimental results. Budak and Ozlu (2007) have proposed an analytical model for the prediction of the stability limit in turning and boring operations. The model provides a multi-directional approach to the dynamic system by solving the stability limit in a matrix form. In addition the true geometry of processes, i.e. the important cutting angles and the insert node radius, are included in the model. Kaymakci et al. (2012) have proposed a unified cutting force model.
models for turning, boring, drilling and milling operations with the inserted tools. The inserted tool tips and their orientations relative to the reference tool coordinates are mathematically represented by ISO tool definition standards. The model can be applied to the cutting forces prediction in turning processes which are similar to boring processes.

However, the entire proposed model of the virtual machining in turning processes focuses on cutting forces, and did not deal with how the individual models are integrated to estimate the final geometries of the turned faces, the geometric deviations and the surface roughness in the virtual machining processes.

Although extensive research has been conducted to develop several models in virtual machining, focusing on development of models for turning processes, However, the virtual machining of parts by considering the physics of the turning processes including kinematic motion deviations of machine tools as relative motion between the tool and workpiece, which are deeply influenced by the geometric deviations of the products, has not yet been clarified.

Yao et al. (2006) have proposed a modeling of virtual workpiece with machining errors representation in turning. By analyzing and modeling the forming process of workpiece and the error sources contributing to machining precision of workpiece in turning operation, the cutter trajectory and attitude representation (CTA_Rep) is presented to characterize the geometric errors of machined workpiece in virtual machining. Then a surface topography simulation model is established to simulate the surface finish profile generated after a turning operation.

Zhu et al. (2007) have proposed research on virtual NC technique in turning and milling process. The geometry simulation and physics simulation are realized by virtual NC technique in turning and milling process. The research demonstrates that the simulations are handled to complete the virtual machining of the product by checking machining ability, rationality of design, prediction manufacture circle, and promptly modifying design.

Anand (2008) has proposed a static error modeling in turning operation and its effect on form errors in his Master’s thesis. This research examines the effect of such errors on part geometry produced by a turning operation. The static errors considered in this research are the machine bed errors usually referred to as 21 geometric errors. These errors are present in the machine in the cold start condition and are induced over a period of time due to gradual wearing and misalignment. This research also includes modeling the spindle error as a significant source of part surface deviation from ideal geometry.

Ramaswami (2010) has proposed an integrated framework for virtual machining and inspection of turned parts in his Ph.D Thesis. The research presented in his dissertation focuses on a two-stage methodology of virtual machining of parts produced on a three-axis turning center and virtual inspection of the produced parts using a bridge-type coordinate measuring machine (CMM). The virtual machining system focuses on a priori predicting the surface profile of the
turned part. The surface profile is generated by modeling the effects of the static errors inherent in the turning center, the error in the spindle motion, machine vibrations, tool geometry, process parameters, and tool wear.

However, the entire proposed model of the virtual machining in turning processes focuses on machining errors and do not deal with how the individual models are integrated to estimate the final geometries of the turned faces which consider both the shape generation motions by considering the machine tool coordinate systems according to ISO 230-1 latest issues, and the cutting tool geometries in accordance with international tool geometry standards.

In this research, the virtual machining in turning processes have been used to simulation model of the machine tools and the machining processes, in order to estimate and to verify the geometric deviations of the turned faces in the machining processes of CNC turning centers and single point tools. The model proposed here represents the turning processes based on both the shape generation motions by considering the machine tool coordinate systems according to ISO 230-1 latest issues, and the cutting tool geometries in accordance with international tool geometry standards.

2.3 Three-Dimensional Surface Roughness

In the present global the competitions, manufacturing processes in industry are developing continuous to increase the quality and the reliability of their products and to decrease the cost of manufacturing process operations. In general it is problematic to determine the values of the process parameters in practice and maximize the manufacturing system performance using the available resources. Benardos and Vosniako (2003) have reviewed and described two main practical problems that engineers face in a manufacturing process. The first is to determine the values of the process parameters that will yield the desired product quality (meet technical specifications) and the second is to maximize manufacturing system performance using the available resources. To overcome these problems, the researchers propose models that try to simulate the conditions during machining and establish cause-effect relationships between various factors and desired product characteristics. Furthermore, the technological advances in the field, for instance the ever-growing use of computer controlled machine tools, have brought up new issues to deal with, which further emphasize the need for more precise predictive models. Surface roughness is a widely used index of product quality and in most cases a technical requirement for mechanical products (Benardos and Vosniako, 2003).

The surface finish of a machined part is one of the most important product quality characteristics. Surface roughness greatly influences the mechanical and physical properties of contacting parts. The understanding of this behaviour is important in many applications such as wear, friction, lubrication, sealing tightness of joint, contact rigidity, contact stress, loaded area
and thermal conductivity. Therefore, engineers are very concerned about the surface roughness of their products (Dong, et al., 1992).

The development of new industries has led to a requirement for super-smooth surfaces and for the ability to measure surfaces of industrial parts accurately and to predict surfaces of parts accurately before real machining processes. There are many researchers working in the area of predicting surface roughness in machining. Benardos and Vosniako (2003) have reviewed and defined four major categories/approaches. These are:

1. Approaches that are based on machining theory to develop analytical models and/or computer algorithms to represent the machined surface (Grzesik, 1996; Lin and Chang, 1998; Baek, et al., 2001);
2. Approaches that examine the effects of various factors through the execution of experiments and the analysis of the results (Abouelatta and Madl, 2001; Ghani and Choudhury, 2002; Shiou and Chuang, 2010);
3. Approaches that use designed experiments (Davim, 2001; Kopac, et al., 2002; Dabnun, et al., 2005; Davidsonga, et al., 2008; Shahrom, et al., 2013);

Keeping in line with objectives of the present research, the literature review will focus on aspects of the approaches that are based on machining theory to develop analytical models and/or computer algorithms to represent the machined surface. The models to estimate the surface roughness has placed emphasis on certain aspects such as process kinematics, cutting tool properties, chip formation mechanism etc. Virtual machining is utilized so as to achieve the goal of building a model that will be able to simulate the creation of the machined surface profile, thus visualizing surface topography and assessing surface roughness. Therefore, several researchers have focused on development of models for machining theory to develop analytical models and/or computer algorithms in order to estimate the surface roughness as discussed below.

Ehmann and Hong (1994, 1995) have proposed a new method to represent the surface generation process, which they called "surface-shaping system" by improving and extending the existing method for modeling machine tool kinematics embodied in the model of the "form-shaping system (FSS)" by Reshetov and Portman (1988). The proposed surface-shaping system model facilitates the evaluation and simulation of the surface texture of machined components by accounting for realistic tool geometry and machine kinematics conditions.

Lee et al. (2001) have focused on prediction of surface roughness and profile in high-speed end milling. A method for simulating the machined surface was presented using the acceleration signal instead of the cutting forces. The argument provided was that the vibration, which is caused by the high speed of the spindle, deteriorates the geometric accuracy of the machined surface. A geometric end milling model was used for modeling the end mill offset and tilt angle.
The computer algorithm was developed in terms of cutting conditions, cutter and workpiece geometry, and runout parameters to determine the angular position of the end mill.

However, the entire research as reported did not include simulation of the 3-dimentional geometry of bored and turned faces in order to estimate 3-dimensional surface roughness including kinematic motion deviations. The cutting tool geometries are mathematically represented by ISO tool definition standards and are important value parameters for estimation of surface roughness.

It is recognized that profiling techniques have been widely used in industry and academic research for manufacturing control and functional control of surface roughness. In some cases, however, the profiling techniques and two-dimensional (2D) parameters defined in standards are inadequate and/or unsuitable for characterizing surfaces. In recent years, there has been an increasing need for the characterization of surface topography in three dimensions (3D). Therefore a clearly defined, effective and widely accepted 3D parameter set is urgently required (Dong, et al., 1994). The estimation of 3D surface roughness of machined parts before real machining process is important for the design and the manufacturing process of products.

Dong et al. (1992, 1994) have proposed a comprehensive study of parameters for characterizing three-dimensional surface topography. The study has discussed the problem of 2D parameter variation through the analysis of 3D surface topography and discussed parameters which are suggested to form the basis of a primary parameter set for characterizing amplitude and some functional properties. In later year, Dong et al. (1995) have discussed reference planes for surface roughness assessment in 3D. From a standardization point of view, reference planes for assessing nominally flat and curved surfaces are recommended through theoretical analysis of algorithms and verification of experimental results.

Bakolas (2003) has proposed a procedure for numerical generation of rough surfaces with prescribed statistical and spectral properties. The procedure extends the method of the linear transformation of matrix, first proposed by employing the non-linear Conjugate Gradient Method (CGM) and Fast Fourier Transforms (FFT) in order to be in position to take into consideration significantly larger portions of the autocorrelation function (ACF), thus improving the agreement between the spectral characteristics of the generated surfaces and the prescribed values. The method is capable of producing arbitrary oriented surfaces by using an ACF that is rotated around its origin. The results show that the method can adequately produce rough surfaces whose statistical and spectral characteristics match the prescribed values.

Quinsat et al. (2008) have proposed a 3D surface roughness parameter that formalizes the relative influence of both machining parameters and surface requirements. The proposed has focused on free-form surface machining which built by considering that during the milling operation, the cutting edges undergo a combined motion of translation and rotation. Simulations serve to define a surface roughness parameter correlated with the machining strategy. Validations
are carried out subsequent to the three-axis milling using a ball end cutter tool of plane surfaces under various cutting conditions via the experimental measurements of surface roughness.

Recently, Buj-Corral et al. (2012) have proposed a surface topography in ball-end milling processes as a function of feed per tooth and radial depth of cut. The numerical model was developed to predict topography and 3D surface roughness based on geometric tool-workpiece intersection. It allows determining surface topography as a function of feed per tooth and revolution, radial depth of cut, axial depth of cut, number of teeth, tool teeth radii, helix angle, eccentricity and phase angle between teeth.

However, the entire proposed model of the estimation of 3D surface roughness did not consider the kinematic motion deviations are integrated to estimate the final geometries of the bored and turned faces. Therefore, the virtual machining of boring and turning processes including kinematic motion deviations has been used to develop simulation model in order to estimate and to verify the 3D surface roughness of the bored and turned faces.

2.4 Three-Dimensional Tolerances

Manufacturing tolerances are intended to determine the intermediate geometrical and dimensional states of a manufactured part during its manufacturing process. Manufacturing tolerances serve to satisfy not only the functional requirements given in the product definition model, but also the manufacturing constraints, such as machine accuracy and minimum extra machining thickness (Zhang and Qiao, 2013). Therefore, manufacturing tolerances have to determine the tolerance analysis in order to estimate the accumulation of the design tolerances on component dimensions and features to ensure that parts will assemble during production. Tolerance analysis is an essential part for mechanical design and manufacturing because it affects not only the performance of products but also the cost.

Over the last thirty years, a large amount of fundamental research efforts has been given to explore the mathematical basis for tolerance analysis. A comprehensive review of dimensioning, tolerancing, and analysis processes may be found in Roy et al. (1991) and Nigam and Tuner (1995). This work will provide an overview of prior research. For tolerance representation, the models or concepts such as: the worst case tolerance analysis (Dantan and Qureshi, 2009; Mansuy, et al., 2011, 2013), statistical approaches to tolerance analysis (Sugimura and Nakamoto, 2003; Rout and Mittal, 2006; Gonzalez and Sanchez, 2009; Beaucaire, et al., 2013), Monte Carlo simulations (Bruyere, et al., 2007; Wu, et al., 2009; Wu, et al., 2012) and computer aided tolerance analysis tool is presented that assists the designer in evaluating worst case quality of assembly after tolerances have been specified (Salomons, et al., 1996a, 1996b; Teissandier, et al., 1999; Chiabert and Orlando, 2004; Anselmetti, et al., 2010).

However, as new generations of tolerancing standards, i.e., ASME Y14.5-2009 and ISO 1101:2012 were released and popularized, geometric tolerances are generally accepted as
industry practices. The traditional one dimensional (1D) or two dimensional (2D) tolerance analysis models are insufficient to meet the ever-tightening and increasingly complex requirements of tolerance analysis in various fields. More specifically, variations of a feature caused by geometric tolerances are three dimensional, which cannot be considered by 1/2D methods. Therefore, researchers and engineers need a new method that can analyze how those geometric tolerances are represented and propagated in three dimensional space urgently (Chen, et al., 2014).

Recently, Chen et al. (2014) have proposed a comprehensive review study of 3D tolerance analysis methods, which reviews four major methods of 3D tolerance analysis and compares them based on the literatures published over the last three decades. The methods studied are Tolerance-Map (T-Map), matrix model, unified Jacobian-Torsor model and direct linearization method (DLM), all of which are research hotspots recently.

Keeping in line with objectives of the present research, the literature review will focus on aspects of the matrix model which uses a displacement matrix to describe the small displacements of a feature within the tolerance zone and the clearance between two features. This model, completed by a set of inequalities defining the bounds of the tolerance zones, reproduces the measurable or non-invariant displacements associated with various types of tolerance. It is very efficient for computation and can easily be integrated into CAD systems. Some of the key papers in the area of 3D tolerance analysis in matrix models are discussed below.

The matrix model introduced by Clement et al. (1991, 1993 and 1994), Desrochers and Rivibret (1997). Clement et al. (1991, 1993 and 1994) have proposed a theory and practice of 3D tolerancing for assembly in CAD/CAM systems which is called Technologically and Topologically Related Surfaces (TTRS) approach. In this approach, tolerances are treated as small displacements. Desrochers and Rivibret (1997) have proposed a matrix approach to the representation of tolerance zones and clearances. The model uses a displacement matrix to describe small displacements of a feature within the tolerance zone and the clearance between two features. The matrix is done using the homogeneous transforms commonly associated with robotic modelling. This matrix representation is completed by a set of inequalities defining the bounds of the tolerance zones.

It is worth mentioning that some computer aided tolerancing software (CATs) based on the matrix model have been developed and applied successfully, such as CATIA.3D FDT (Prisco and Giorleo, 2002), FROOM (Salomons, et al., 1996a, 1996b) and the Quick GPS (Geometrical Product Specification) system has been developed in a CATIA V5 (Anselmetti, et al., 2010).

The statistical method of the matrix model for tolerance analysis can be found in Whitney et al. (1994). Sugimura et al. (2003) and Satonaka et al. (2005a, 2005b, 2007a) have proposed a study of statistical analysis of 3D geometric tolerances of products. The objective of the present research is to develop a systematic method for planning and analyzing geometric tolerances of
three-dimensional mechanical products. The analyses of the geometric deviations of the target features against the root datum features are carried using the Monte-Carlo simulation method. The 3D geometric tolerances are estimated using the statistical deviations of the positions and the orientations of the geometric features, based on the deviation parameters and the relationships between the datum features and the target features. In particular, emphasis is given to the analysis of the statistical deviations of the geometric features under the MMC (Maximum Material Conditions), which specifies the interaction between the dimensional tolerances and the geometric tolerances. As regards the dimensional tolerances and the geometric tolerances, aimed at realizing systematic analysis and design methodologies for the three dimensional products as mentioned above. The analyses of kinematic motion deviations of machining centers based on geometric tolerances have not yet been discussed.

Satonaka et al. (2007b, 2008) have applied and developed the mathematical analysis of 3D geometric tolerance from Sugimura et al. (2003) to the analysis of kinematic motion deviations of 3-axis machining centers based on geometric tolerances. The geometric deviations of the guide-ways were discussed, and a method was proposed to estimate the standard deviations of the geometric deviations of the guide-ways. The geometric deviations of the linear tables were formulated, based on the priorities among the guide-ways of the linear tables. However, the latest research work has not yet discussed the kinematic motion deviations based on geometric tolerance of the rotary tables which are not linear against the rotational angles of the tables.

In this research, mathematical models representing the kinematic motion deviations of the rotary tables and the 5-axis machining centers are established. The models are based on the 3D geometric tolerances of the components, and estimation of 3D tolerances from the virtual machining of the CNC machining centers in boring processes and the CNC turning centers in turning processes including kinematic motion deviations.
Chapter 3

Simulation of Virtual Machining Systems in Boring and Turning Processes
3. Simulation of Virtual Machining Systems in Boring and Turning Processes

3.1 Introduction

As discussed in Section 1.3, this chapter focuses on the part manufacturing processes in a virtual manner by incorporating the effects of various factors of machining processes. A virtual machining system described below represents virtual machining processes of the boring process on CNC machining centers and turning process on CNC turning centers, and simulated parts are generated taking into consideration of kinematic motion deviations due to spindle motion errors and geometric errors of inserted cutting tools. The parts thus generated can be viewed as a representative of an actual part machined on that machine and can be used to evaluate the 3D surface roughness and tolerance on the geometry of the part. Figure 3-1 illustrates the important components of the virtual machining system in the context of the overall system.

![Diagram of Virtual Machining System](image)

Fig. 3-1 Key components of the virtual machining system

The key components of the virtual machining system in this chapter are:
- Modeling of shape generation motions
- Machine tool coordinate systems, position and orientation errors (in accordance with ISO 230-1:2012)
• Kinematic motion deviations in boring processes
• Kinematic motion deviations in turning processes
• Generation of simulated parts

Details of these models are provided below:

3.2 Modeling of Shape Generation Motions

Figure 3-2 summarizes the factors which affect the machining errors of the product surfaces and their relationships from the viewpoint of the shape generation processes of the machine tools. The geometric errors of the product surfaces are determined mainly from the relative motion errors between the tools and the workpieces, the geometric errors of the tools, and the errors in the material removal processes. The requirements of the geometric accuracy of the product surfaces may not be satisfied due to the factors originated from the structural design and the manufacturing processes of all the components of the machine tools. The relations among the factors shown in Fig. 3-2 are very complicated, and are not yet clear (Sugimura and Murabe, 1997). Therefore, this section deals mainly with the kinematic motion errors in relative motions between the tools and workpieces.

Fig. 3-2 Factors affecting machining errors (Sugimura and Murabe, 1997)

The shape generation processes of the machine tools are generally represented by the shape generation motions, and also the tool geometries (Reshetov and Portman, 1988; Sugimura, et al., 1981). The shape generation motions, which are the relative motions of the tools against the workpieces, are executed by a set of rigid component of the machine tools. Figure 3-3 shows a schematic illustration describing the rigid components called units element and the shape generation motions among them. Figure 3-4 shows chain-link diagram representing the shape generation motions; $S_0, S_1, \ldots, S_l$ represent the links, $S_0$ is the part to be machined, $S_l$ is the cutting tool, and $S_1, \ldots, S_{l-1}$ are the intermediate links; $q_0, q_1, \ldots, q_l$ are motion parameters of the units; $k_1, k_2, \ldots, k_l$ are the relative motion of the units.
3.2.1 Homogeneous point of coordinates

Vectors of fourth order of points in a three-dimensional space are divided into two classes depending upon their geometrical (physical) natures: eigenvectors and non-eigenvectors. Point radius vectors belong to the first category, and vectors obtainable by differentiation or infinitely small transformation of radius vectors belong to the second category. The coordinates of the vectors of the fourth order are called homogeneous coordinates. Eigenvectors have a fourth homogeneous coordinate equal to unity, and non-eigenvectors have a fourth homogeneous coordinate equal to zero.

The radius vectors of the points with Cartesian coordinates \( x, y, z \) are written in the form of a column vector of the fourth order.
Furthermore, in order to save space, we shall widely use the representation of the column in transposition form, i.e., in a form of a row vector

\[
\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}
\]

(3-1)

where \( T \) is the sign of transposition.

Vector \( r \) can be represented as

\[
r = xe^1 + ye^2 + ze^3 + le^4,
\]

where \( e^1, e^2, e^3 \) are unit vectors of the axes of the coordinates;

\[
e^1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad e^2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad e^3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.
\]

\( e^4 \) is the radius vector of the origin of the coordinates;

\[
e^4 = (0, 0, 0, 1)^T.
\]

(3-2)

The only operation that transforms the radius vector into another radius vector is multiplication of the radius vector by the matrix of transformation of the coordinates. By definition of the radius vector, its origin always coincides with the origin of the coordinates (Reshetov and Portman, 1988).

### 3.2.2 Transformation of coordinates

Transformation of coordinates, consider two systems of coordinates \( S_{i-1} \) and \( S_i \). The same point in space has different coordinates in these systems with the exception of the trivial case when \( S_{i-1} \) and \( S_i \) completely coincide. Denote by \( X_0 \) and \( X_l \) the radius vectors of the type Eq. (3-1) of a point in the two-coordinate systems, they are linked by the matrix relation

\[
X_0 = A_l(q_1, q_2, ..., q_l)X_l
\]

(3-3)

where,
\( X_t \): Position vector of a point on the tool given in the tool coordinate system,
\( X_0 \): Position vector of the point given in the workpiece coordinate system, and
\( A_{ij} \): The coordinate transformation matrix of order 4 x 4 having the following structure:

\[
A = \begin{bmatrix}
    a_{11} & a_{12} & a_{13} & a_{14} \\
    a_{21} & a_{22} & a_{23} & a_{24} \\
    a_{31} & a_{32} & a_{33} & a_{34} \\
    0 & 0 & 0 & 1
\end{bmatrix},
\]

(3-4)

in which the upper left 3 x 3 block describes the rotation of system \( S_i \) relative to its origin of coordinates \( O_i \) to such a position that the axes of systems \( S_{i-1} \) and \( S_i \) become parallel and similarly directed. Hence, it follows that this block represents an orthogonal matrix, i.e., for all \( i, k = 1, 2, 3 \),

\[
\sum_{j=1}^{3} a_{ij} a_{kj} = \sum_{j=1}^{3} a_{jk} a_{kj} = \begin{cases} 0, & \text{if } i \neq k, \\ 1, & \text{if } i = k, \end{cases}
\]

and, furthermore, the determinant of this matrix is

\[
\begin{vmatrix}
    a_{11} & a_{12} & a_{13} \\
    a_{21} & a_{22} & a_{23} \\
    a_{31} & a_{32} & a_{33}
\end{vmatrix} = 1.
\]

The first three coordinates \( a_{14}, a_{24} \) and \( a_{34} \) of the fourth column of matrix \( A \) are Cartesian coordinates of the origin \( O_i \) of system \( S_i \) in system \( S_{i-1} \).

Let systems \( S_{i-1} \) and \( S_i \) be tied with two consecutive shape generation links that the relative motions of the links are limited to six of the simplest motions, formula Eq. (3-3) can be represented as:

\[
A_{ij}(q_1, q_2, ..., q_I) = \prod_{i=1}^{J} A_{ij}^{\alpha}(\beta_i)
\]

(3-5)

where,

\( A_{ij}^{\alpha}(\beta) \): 4x4 transformation matrices representing kinematic motions and positions
\( \alpha(=1, ..6) \): Indices representing the directions of kinematic motion and positions. 1, 2 and 3 means linear motions and positions of x, y and z directions, respectively. 4, 5 and 6 means rotary motions and positions around x, y and z directions, respectively.
\( \beta \): Values of kinematic motions and positions
\( \gamma \): Indices representing coordinate systems
The individual relative motions given by $A^\alpha$ are generally either the translation motions along the X, Y and Z-axis or the rotary motions around the X, Y and Z-axis. The transformation matrices are described by the matrices shown in Table 3-1.

Table 3-1 Transformation matrices describing translation motions and rotary motion  
(Sugimura and Murabe, 1997)

<table>
<thead>
<tr>
<th>Motion Type</th>
<th>Axis</th>
<th>Relative Motion</th>
<th>Transformation Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>Translation Motion</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
|                   | X    | 1               | $A^1 = 
\begin{bmatrix}
1 & 0 & 0 & x \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}$ |
|                   | Y    | 2               | $A^2 = 
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & y \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}$ |
|                   | Z    | 3               | $A^3 = 
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & z \\
0 & 0 & 0 & 1
\end{bmatrix}$ |
| Rotary Motion     |      |                 |                       |
|                   | A    | 4               | $A^4 = 
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & cos \phi & -sin \phi & 0 \\
0 & sin \phi & cos \phi & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}$ |
|                   | B    | 5               | $A^5 = 
\begin{bmatrix}
cos \psi & 0 & sin \psi & 0 \\
0 & 1 & 0 & 0 \\
-sin \psi & 0 & cos \psi & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}$ |
|                   | C    | 6               | $A^6 = 
\begin{bmatrix}
cos \theta & -sin \theta & 0 & 0 \\
sin \theta & cos \theta & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}$ |
3.2.3 Homogeneous transformation matrix (HTM) of rotational and translational errors

To study the error behaviour of a machine movement, the kinematic model of the machine is developed in the form of a series of HTM. In this model, each type of errors is defined and analysed by using the HTM model to determine the effect of the errors on the cutting-point positional accuracy with respect to the workpiece. Consider the simple case of a linear axis motion, the carriage moves along the Y-axis to a point \((0, b, 0)\), as shown in Fig. 3-5.

![Fig. 3-5 Linear motion carriage in Y-axis (Nerdnoi, 1999)](image)

The \(N'-\)frame is defined as pure translation movement and the origin point of \(N'-\)frame is defined as point \(O_{N'}\) which is offset \((0, b, 0)\) from the origin point of \(R\)-frame. The HTM of the \(N'-\)frame with respect to the \(R\)-frame is the HTM of \(A^2\) in Table 3-1, the result is:

\[
A^2_{RN'} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & b \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix} \tag{3-6}
\]

For any linear motion, a rigid body will always have 6 error components which consist of three rotations \((\theta_x, \theta_y, \theta_z)\) and three translations \((\delta_x, \delta_y, \delta_z)\), see Fig. 3-6. To understand these errors, the \(N''\)-frame and the \(N\)-frame are defined where the \(N''\)-frame is pure translation respect to the \(N'\)-frame and has an offset of \((\delta_x, \delta_y, \delta_z)\) from the origin of the \(N'\)-frame. The \(N\)-frame is pure rotation with respect to the \(N''\)-frame and its origin point is on the origin point of the \(N''\)-frame. The HTM of pure translation can be found by multiplying equations in Table 3-1 in transformation matrix of \(A^1\), \(A^2\) and \(A^3\) as show the Eq. (3-7). The HTM of three rotations can be
found by multiplying equations in Table 3-1 in transformation matrix of $A^4$, $A^5$ and $A^6$, as shown in the Eq. (3-8), and $E_{ij}$ is the infinitesimal matrix of the position errors of the $i$th link.

\[
\begin{bmatrix}
1 & 0 & 0 & \delta_x \\
0 & 1 & 0 & \delta_y \\
0 & 0 & 1 & \delta_z \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  

(3-7)

\[
E_{NN'} = \begin{bmatrix}
c_{\theta_x}c_{\theta_y} & c_{\theta_x}s_{\theta_y}s_{\theta_z} - s_{\theta_x}c_{\theta_z} & c_{\theta_x}s_{\theta_y}c_{\theta_z} + s_{\theta_x}s_{\theta_z} & 0 \\
\theta_x & s_{\theta_x}s_{\theta_y}s_{\theta_z} + c_{\theta_x}c_{\theta_z} & s_{\theta_x}s_{\theta_y}c_{\theta_z} - c_{\theta_x}s_{\theta_z} & 0 \\
- s_{\theta_y} & c_{\theta_y}s_{\theta_z} & c_{\theta_y}c_{\theta_z} & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  

(3-8)

where: $c_{\theta_i} = \cos \theta_i$; $s_{\theta_i} = \sin \theta_i$; $c_{\theta_i} = \cos \theta_i$; $s_{\theta_i} = \sin \theta_i$; $c_{\theta_i} = \cos \theta_i$; $s_{\theta_i} = \sin \theta_i$
Because the angles of these rotating errors are very small, most are in minutes of arc, these approximate values can be adopted:

\[
c\theta_x c\theta_y = c\theta_x c\theta_z = c\theta_x c\theta_x = 1;
\]

\[
s\theta_x c\theta_y = s\theta_x c\theta_z = \theta_x = \varepsilon_z;
\]

\[
c\theta_x s\theta_y = c\theta_x s\theta_z = \theta_x = \varepsilon_x;
\]

\[
c\theta_x s\theta_z c\theta_z = s\theta_y = \theta_y = \varepsilon_y;
\]

\[
c\theta_x s\theta_z s\theta_z = s\theta_y s\theta_x s\theta_z = s\theta_x s\theta_x c\theta_z = 0;
\]

So, the HTM for those six error parameters in Eq. (3-8) is:

\[
E_{NN} = \begin{bmatrix}
1 & -\varepsilon_z & \varepsilon_y & \delta_x \\
\varepsilon_z & 1 & -\varepsilon_x & \delta_y \\
-\varepsilon_y & \varepsilon_x & 1 & \delta_z \\
0 & 0 & 0 & 1
\end{bmatrix}
\] (3-9)

These 6 error parameters of the linear movement that is shown in Eq. (3-9) are known as:

\(\delta_x\): Position error, with deflection in the direction of feed

\(\delta_y\): Straightness error in X-axis direction

\(\delta_z\): Straightness error in Z-axis direction

\(\varepsilon_y\): Angular error about the Y-axis (Roll movement; rotation about the feed axis)

\(\varepsilon_x\): Angular error about the X-axis (Pitch or Tip movement; rotation about one axis in the plane of the table perpendicular to the feed axis)

\(\varepsilon_z\): Angular error about the Z-axis (Yaw or Rotary movement; rotation normal to the plane of the table).

The HTM of the N-frame respect to the R-frame for the linear movement in the Y-axis to point \((0, b, 0)\) which include all errors is:

\[
X_R = A_{RN}^2(b)E_{NN}X_N
\]

\[
X_{RN_{-actual}} = \begin{bmatrix}
1 & -\varepsilon_z & \varepsilon_y & \delta_x \\
\varepsilon_z & 1 & -\varepsilon_x & b + \delta_y \\
-\varepsilon_y & \varepsilon_x & 1 & \delta_z \\
0 & 0 & 0 & 1
\end{bmatrix}
\] (3-10)

These parameters are functions of the positions of the carriage in the reference frame. By measuring these parameters along the moving axis, the actual position of any point on the carriage frame (N-frame) can be transformed into the reference frame (R-frame) (Nerdnoi, 1999).
Six of these parameters which can be used the kinematic motion deviation for simulation in the virtual machining is given randomly, normal distribution.

### 3.3 Machine Tool Coordinate System, Position and Orientation Errors

Although the nomenclature for the machine axes of motion is defined ISO 841, mere labelling of the axes is not adequate for the purposes of the characterization of machine geometric errors and the compensation for their effects using machine tool controllers. This section topic provides a systematic way to define a machine tool coordinate system in accordance with ISO 230-1:2012 in order to generate the virtual machining, in which the geometric relationships are among the axes of motion and associated errors.

By describing the positions and orientations of the axes of motion in a coordinate system, the alignment errors among these axes, necessary for geometric accuracy evaluation and/or software-based error compensation, can be identified for any machine structural configuration.

To eliminate redundant alignment error measurements among the axes of motion, the origin and orientation of the machine tool coordinate system is chosen to orientate with the primary axis of motion (defining two orientations) and the secondary axis of motion (defining the third orientation) (ISO 230-1:2012).

In this research, the proposed machine tool coordinate system is the coordinate systems on 3-axis machining centers of boring processes and CNC turning centers of turning processes

#### 3.3.1 Machine tool coordinate system of boring processes

The boring processes on 3-axis machining centers are carried out by both the spindle rotations and the linear feed motion along Z-axis, therefore, the motion deviations of the spindle rotation is an important issue for evaluating the geometric deviations of bored holes. The coordinate systems for the boring processes are set according to ISO 230-1:2012 by considering in the reference straight line of the linear axis of motion and axis the rotation.

As prescribed by ISO 841, the machine coordinate system is a right-hand rectangular system. The position and orientation of the machine tool coordinate system is typically defined by the axes of motion of its moving components.

The coordinate systems for the boring processes are shown in Fig. 3-7. The reference coordinate system $O_R$ is set according to ISO 230-1:2012, and the other coordinate systems are as follows.

1) The coordinate system $O_S$ of the spindle is fixed on the spindle and has four deviation parameters representing the position and orientation deviations of the average rotational axis
of the spindle. They are \( \delta_x(C) \), \( \delta_y(C) \), \( \epsilon_x \) and \( \epsilon_y \) which represent the positioning deviations in X- and Y-axis and the rotational deviations around X- and Y-axis.

2) The coordinate system \( O_B \) of the boring bar is fixed on the boring bar and the parameter \( z_B \) gives the distance between \( O_s \) and \( O_B \) representing the length of the boring bar.

3) The coordinate system \( O_T \) of the tool tip is fixed on the tool tip and the parameter \( R_T \) gives the distance between \( O_B \) and \( O_T \) representing the radial position of the tool tip against the boring bar axis.

4) The coordinate system \( O_E \) of the cutting edge is fixed on the cutting edge and the parameters \( R_E \) and \( z_E \) gives the distance between \( O_T \) and \( O_E \) representing the shapes and the dimensions of the tool tip.

Fig. 3-7 Coordinate systems for boring processes
3.3.2 Machine tool coordinate system of turning processes

The turning processes on CNC turning centers are carried out by the spindle rotations, the linear feed motion along the Z-axis and the depth of cut along the X-axis, therefore, the motion deviations of the spindle rotation, feed motion and depth of cut are important issues for evaluating the geometric deviations of turned parts. The coordinate systems for the turning processes are set according to ISO 230-1:2012 by considerations described for the reference straight line of the linear axis of motion and axis of the rotation.

The coordinate systems for the turning processes on CNC turning centers are shown in Fig. 3-8. The coordinate systems $O_S$, $O_R$, $O_{CR}$, $O_{CS}$, $O_T$ and $O_E$ are set according to ISO 230-1:2012. The turning processes are carried out by the spindle rotations, the linear feed motion along Z-axis and the depth of cut along X-axis, and the other coordinate systems are as follows.

1) The coordinate system $O_S$ of the spindle is fixed on the spindle and has two deviation parameters representing the orientation deviations of the average rotational axis of the spindle. They are $\varepsilon_x$ and $\varepsilon_y$ which represent the rotational deviations around X- and Y-axis.

2) The coordinate system $O_{CR}$ of the carriage is fixed on the middle and below the carriage and the parameter $z$ gives the distance between $O_S$ and $O_{CR}$ representing the length of the moving carriage along Z-axis.

3) The coordinate system $O_{CS}$ of the cross slide is fixed on center of the cross slide, the parameter $x$ gives the distance between $O_{CR}$ and $O_{CS}$ representing the length of the moving cross slide along X-axis, the parameter $L_x$ and $L_z$ representing the length of the cross slide axis against the tool tip.

4) The coordinate system $O_E$ of the cutting edge is fixed on the cutting edge and the parameters $x_E$ and $z_E$ gives the distance between $O_T$ and $O_E$ representing the shapes and the dimensions of the tool tip.

(a) Structures of CNC turning centers
3.4 Kinematic Motion Deviations in Boring Processes

3.4.1 Shape generation motions in boring processes

The boring processes on CNC machining centers are carried out by both the spindle rotations and the linear feed motion along Z-axis, therefore, the motion deviations of the spindle rotation is an important issue for evaluating the geometric deviations of bored holes.

![Diagram of coordinate systems and their deviations](image)

**Fig. 3-9 Coordinate systems and their deviations of boring processes**

Five cartesian coordinate systems shown in Fig. 3-9 are set to represent the kinematic motion deviations which are in accordance with Fig. 3-7. They are, $O_R$, $O_S$, $O_B$, $O_T$ and $O_E$ which represent the coordinate systems of the reference, spindles, boring bars, tool tips and cutting edges, respectively.
The shape generation processes of the machine tools are generally represented by the shape
generation motions, and also the tool geometries. The shape generation motions, which are the
relative motions of the tools against the workpieces, are executed by a set of rigid component of
the machine tools (Sugimura and Murabe, 1997). The shape generation motions of the cutting
edge against the reference are described by Eq. (3-11).

\[
X_R = A_{RS}A_{SB}A_{BT}A_{TE}X_E
\]  

(3-11)

where,

\( A_{ij} \): 4 by 4 homogeneous transformation matrices representing the relative positions and the
kinematic motions between pairs of rigid bodies \( i \) and \( j \)

\( X_E \): Position vector of a point on the cutting edge in the cutting edge coordinate system.

\( X_R \): Position vector of the point on the cutting edge in the reference coordinate system.

The individual matrices include some kinematic and position deviations due to both the
motion errors and the set-up errors, therefore, the kinematic motion deviations in the boring
processes discussed in the following equation is obtained from Eq. (3-11) by applying the
parameters mentioned above.

\[
X_R = E_z^{-1}A^3(\varepsilon)^{-1}E_{RS}A^6_{RS}(\theta_{\varepsilon})E_{\theta}A^3_{SB}(-z_B)A^2_{BT}(R_T)A^2_{TE}(-y_E)A^3_{TE}(-z_E)X_E
\]  

(3-12)

where,

\( E_z \): Positioning deviations and straightness deviations in Z-axis

\( E_{RS} \): Parallelism errors of the spindle to the reference coordinate system

\( E_{\theta} \): Position and rotational deviations of spindles

\( \theta_{\varepsilon} \): Rotational angle of spindle

\( z \): Machine positions along Z-axis

\( z_B \): Length along the Z-axis of the boring bar

\( R_T \): Radial position of tool tip against the boring bar axis

\( y_E \): Distance between \( O_T \) and \( O_E \) of the tool tip along the Y-axis

\( z_E \): Distance between \( O_T \) and \( O_E \) of the tool tip along the Z-axis

In the present research, it is assumed that the parallelism errors and the squareness errors are
zero, in order to simplify the analysis.

As regards to the kinematic motion deviations, \( E_{\theta} \) and \( E_z \) are given in the following equations.

\[
E_{\theta} = \begin{bmatrix}
1 & 0 & \varepsilon_y & \delta_y(C) \\
0 & 1 & -\varepsilon_x & \delta_x(C) \\
-\varepsilon_y & \varepsilon_x & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  

(3-13)
where,
\[ \varepsilon_x : \text{Angular position deviation of X-axis in spindle axis C} \]
\[ \varepsilon_y : \text{Angular position deviation of Y-axis in spindle axis C} \]
\[ \delta_x(C) : \text{Positioning deviation of X-axis in spindle axis C} \]
\[ \delta_y(C) : \text{Positioning deviation of Y-axis in spindle axis C} \]

\[
E_z = \begin{bmatrix}
1 & 0 & 0 & \delta_z(z) \\
0 & 1 & 0 & \delta_y(z) \\
0 & 0 & 1 & \delta_x(z) \\
0 & 0 & 1 & 1 \\
\end{bmatrix} \quad E_x = \begin{bmatrix}
1 & 0 & 1 & \delta_x(x) \\
0 & 1 & 0 & \delta_y(x) \\
0 & 0 & 1 & \delta_z(x) \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

(3-14)

where,
\[ \delta(z), \delta(x) : \text{Positioning deviations} \]
\[ \delta_l(z), \delta_l(x), \delta_l(x) : \text{Straightness deviations} \]

In the present research, it is assumed that the positioning errors and the straightness errors in X-axis are zero, since the motions in the X-axis are fixed in the boring processes.

3.4.2 Formulation of kinematic motions in boring processes

3.4.2.1 Spindle motion

The spindle motion is represented by the following equation.

\[
A_{RS, actual} = E_z^{-1} A_3(z) A_6^6(\theta_z) E_\theta \\
= \begin{bmatrix}
1 & 0 & 0 & -\delta_z(z) \\
0 & 1 & 0 & -\delta_y(z) \\
0 & 0 & 1 & -\delta_x(z) \\
0 & 0 & 0 & 1 \\
\end{bmatrix} \begin{bmatrix}
\cos \theta_z & -\sin \theta_z & 0 & 0 \\
\sin \theta_z & \cos \theta_z & 0 & 0 \\
\cos \theta_z & -\sin \theta_z & 0 & 0 \\
\sin \theta_z & \cos \theta_z & 0 & 0 \\
\end{bmatrix} \begin{bmatrix}
1 & 0 & 0 & \varepsilon_y \\
0 & 1 & 0 & \varepsilon_x \\
0 & 0 & 1 & -\varepsilon_y \\
0 & 0 & 0 & 1 \\
\end{bmatrix} \\
= \begin{bmatrix}
\cos \theta_z & -\sin \theta_z & \varepsilon_y \cos \theta_z + \varepsilon_x \sin \theta_z & \delta_x(C) \cos \theta_z - \delta_y(z) - \delta_y(C) \sin \theta_z \\
\sin \theta_z & \cos \theta_z & \varepsilon_y \sin \theta_z - \varepsilon_x \cos \theta_z & \delta_y(C) \cos \theta_z - \delta_y(z) + \delta_x(C) \sin \theta_z \\
0 & 0 & 1 & -\delta_z(z) - z \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

(3-15)

3.4.2.2 Boring bar motion

The motion of boring bar is represented by the following equation.

\[
A_{SB, actual} = [Eq.(3-15)] A_3^R(-z_B) 
\]
where,

\[ f_1 = \delta_x(C) \cos \theta_z - \delta_y(z) - \delta_y(C) \sin \theta_z - z_B(\varepsilon_y \cos \theta_z + \varepsilon_x \sin \theta_z) \]

\[ f_2 = \delta_x(C) \cos \theta_z - \delta_y(z) + \delta_y(C) \sin \theta_z + z_B(\varepsilon_y \cos \theta_z - \varepsilon_x \sin \theta_z) \]

### 3.4.3 Formulation of kinematic motions of cutting edge in boring processes

Generalized geometric models of inserted cutters have been proposed by Kaymakci et al. (2012) using mathematical modeling. An approach similar to that of Engin and Altintas (2001) has been used containing certain modifications and improvements. Firstly, using the inputs, a mathematical model of one insert was developed on the local coordinate system positioned at the cutting reference point of the insert. Figure 3-10 illustrates the local coordinate system and the cutting reference point on two different inserts. The aim of this model is to calculate the control points that are sufficient to define the features (nose radius, corner chamfer, wiper edge etc.) on an insert (Kaymakci, 2009).

There are various inserted tool tip geometries that can be used in the cutting tools, and seventeen inserted tool tip shapes are defined in ISO 13399 standards (ISO 13399:2006). The generalized geometric model of the inserted tools is shown in Fig. 3-11, starting with the placement of the tool tips on the cutter bodies, the identification of oblique tool angles needed in the cutting mechanics model and the kinematics of cutting operations. The geometry of the tool tips are defined in their local coordinate system \( O_T \) analytically.
The control points are derived as functions of insert parameters that have been adapted from Kaymakci et al. (2012). The coordinates of the control points are given as follows based on the standardized tool tip geometries.

\[
\text{CRP}(x, y, z) = (0, 0, 0)
\]
\[
A(x, y, z) = (0, -b_s + r_e (\cot \kappa_r - \csc \kappa_r), 0)
\]
\[
B(x, y, z) = (0, r_e (\cot \kappa_r - \csc \kappa_r), 0)
\]

Fig. 3-10 Cutting reference points on different inserts (Kaymakci, 2009)

Fig. 3-11 Tool tips and cutting edges of boring processes (Adapted from Kaymakci, et al., 2012)
\[ C(x,y,z) = (0, r_c(cot \kappa - csc \kappa), r_c) \]
\[ D(x,y,z) = (0, r_c cos \kappa, tan \frac{\kappa_r}{2}, r_c(1 - csc \kappa)) \]
\[ E(x,y,z) = (0, A_y + L cos \kappa_r - cos(\epsilon_r + \kappa_r) csc \epsilon_r(b_z sin \kappa_r - (r_c cos \kappa_r - 1)), \]
\[ L sin \kappa_r - csc \epsilon_r sin(\epsilon_r + \kappa_r)(b_z sin \kappa_r - (r_c cos \kappa_r - 1))) \]
\[ O_T(x,y,z) = (0, A_y + \frac{1}{2}(L cos \kappa_r + iw cos(\epsilon_r + \kappa_r) - 2 cos(\epsilon_r + \kappa_r) csc \epsilon_r(b_z sin \kappa_r - r_c(\epsilon_r + \kappa_r - 1)))) \]
\[ \frac{1}{2}(L sin \kappa_r + iw sin(\epsilon_r + \kappa_r) - 2 csc \epsilon_r sin(\epsilon_r + \kappa_r)(b_z sin \kappa_r - r_c(\epsilon_r + \kappa_r - 1))) \]

(3-18)

where,

**CRP** : Cutting reference point
**A, B, C, D and E** : Control points of cutting edge
**\( \epsilon_r \)** : Tool included (nose) angle
**iw** : Insert width
**L** : Insert length
**\( \kappa_r \)** : Cutting edge angle of the insert
**b_z** : Wiper edge length
**r_c** : Corner radius

Therefore, the motions of the cutting edge in tool tips are represented by the following equation based on Eq. (3-18) and multiplied with Eq. (3-17).

\[ A_{TE, actual} = [Eq. (3-17)] A_{TE, actual}^2 A_{TE, actual}^3 A_{TE, actual}^4 A_{TE, actual}^5 \]
\[ y_E = A_y + \frac{1}{2}(L cos \kappa_r + iw cos(\epsilon_r + \kappa_r) - 2 cos(\epsilon_r + \kappa_r) csc \epsilon_r(b_z sin \kappa_r - r_c(\epsilon_r + \kappa_r - 1))) \]
\[ z_E = \frac{1}{2}(L sin \kappa_r + iw sin(\epsilon_r + \kappa_r) - 2 csc \epsilon_r sin(\epsilon_r + \kappa_r)(b_z sin \kappa_r - r_c(\epsilon_r + \kappa_r - 1))) \]

\[ A_{TE, actual} = \begin{bmatrix}
\cos \theta_z & -\sin \theta_z & \epsilon_y \cos \theta_z + \epsilon_z \sin \theta_z & f_3 \\
\sin \theta_z & \cos \theta_z & \epsilon_y \sin \theta_z - \epsilon_z \cos \theta_z & f_4 \\
-\epsilon_y & \epsilon_z & 1 & \epsilon_x R_T - z_B - z \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[ 0 0 0 1 \]

(3-19)
The kinematic motions of the cutting edge of boring processes against the reference coordinate system $O_R$ are presented by Eq. (3-20) including the kinematic deviations.

$$
P_{x,Boring} = \delta_x(C) \cos \theta_z - F_2(F_3 + F_4 - F_6) - \delta_y(z) - \delta_y(C) \sin \theta_z - R_T \sin \theta_z + \sin \theta_z F_5 - z_B F_2
$$

$$
P_{y,Boring} = F_1(F_3 + F_4 - F_6) - \delta_y(z) + \delta_y(C) \cos \theta_z + R_T \cos \theta_z + \delta_x(C) \sin \theta_z - \cos \theta_z F_5 + z_B F_1
$$

$$
P_{z,Boring} = \varepsilon_x R_T - z_B - z - \varepsilon_z F_5 - F_3 - F_4 - \delta_z(z) + F_6
$$

(3-20)

where,

$$
F_1 = \varepsilon_x \cos \theta_z - \varepsilon_y \sin \theta_z
$$

$$
F_2 = \varepsilon_y \cos \theta_z + \varepsilon_x \sin \theta_z
$$

$$
F_3 = \frac{i \omega \sin(\varepsilon_r + \kappa_r)}{2}
$$

$$
F_4 = \frac{L \sin \kappa_r}{2}
$$

$$
F_5 = \frac{i \omega \cos(\varepsilon_r + \kappa_r)}{2} - b_s + \frac{L \cos \kappa_r}{2} + \epsilon_s \left( \cot \kappa_r - \frac{1}{\sin \kappa_r} \right) - \frac{\cos(\varepsilon_r + \kappa_r) E_7}{\sin \epsilon_r}
$$

$$
F_6 = \frac{\sin(\varepsilon_r + \kappa_r) E_7}{\sin \epsilon_r}
$$

$$
F_7 = b_s \sin \kappa_r - r_s (\cos \kappa_r - 1)
$$

### 3.5 Kinematic Motion Deviations in Turning Processes

#### 3.5.1 Shape generation motions in turning processes

The turning processes on CNC turning centers are carried out by the spindle rotations, the linear feed motion along the Z-axis and depth of cut along the X-axis. Therefore, the motion deviations of the spindle rotation are an important issue for evaluating the geometric deviations of turned parts.

Six cartesian coordinate systems shown in Fig. 3-12 are set to represent the kinematic motion deviations which are in accordance with Fig. 3-8. They are $O_R$, $O_S$, $O_{CR}$, $O_{CS}$, $O_T$ and $O_E$ which represent the coordinate systems of the reference, spindles, carriages, cross slides, tool tips and cutting edges, respectively.
The shape generation processes of the machine tools are generally represented by the shape generation motions, and also the tool geometries. The shape generation motions, which are the relative motions of the tools against the workpieces, are executed by a set of rigid component of the machine tools (Sugimura and Murabe, 1997). The shape generation motions of the cutting edge against the reference are described by Eq. (3-21).

\[ X_S = (A_{RS})^{-1}A_{RCR}A_{CRCS}A_{CST}A_{TE}X_E \]  

(3-21)

The individual matrices include some kinematic and position deviations due to both the motion errors and the set-up errors, therefore, the kinematic motion deviations in the turning processes are discussed in the following equation is obtained from Eq. (3-21) by applying the parameters mentioned above.

\[ X_S = E_{z}^{-1}A_{RS}^{-1}E_{RS}^{-1}A_{RCR}^{-1}(z)E_{z}E_{CRCS}A_{CRCS}(x)E_{z}A_{CST}^{-1}(-L_x)A_{CST}^{-1}(-L_z)A_{TE}^{-1}(-x_E)A_{TE}^{-1}(z_E)X_E \]  

(3-22)

where,

- \( E_{z} \): Positioning deviations and straightness deviations in Z-axis
- \( E_{RS} \): Parallelism errors of the spindle to the reference coordinate system
- \( E_{\theta} \): Position and rotational deviations of spindles
- \( \theta_z \): Rotational angle of spindle
- \( z \): Machine positions along Z-axis
- \( z_{th} \): Length along the Z-axis of the boring bar
- \( R_T \): Radial position of tool tip against the boring bar axis
- \( y_{E} \): Distance between \( O_T \) and \( O_E \) of the tool tip along the Y-axis
- \( z_{E} \): Distance between \( O_T \) and \( O_E \) of the tool tip along the Z-axis

(The items mentioned above are needs in Eq. (3-22) as Eq. (3-12))

- \( x \): Machine positions along X-axis
- \( x_{E} \): Distance between \( O_T \) and \( O_E \) of the tool tip along the X-axis
- \( L_x, L_z \): Distance between \( O_{CS} \) and \( O_T \)
- \( E_{x} \): Positioning deviations and straightness deviations in X-axis
- \( E_{CRCS} \): Squareness errors between the Z-axis and X-axis.
In the present research, it is assumed that the parallelism errors and the squareness errors are zero, in order to simplify the analysis.

As regards to the kinematic motion deviations, $E_\theta$ and $E_x$ are given in the following equations.

\[
E_\theta = \begin{bmatrix}
1 & 0 & \varepsilon_y & \delta_x(C) \\
0 & 1 & -\varepsilon_x & \delta_y(C) \\
-\varepsilon_y & \varepsilon_x & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  \hspace{1cm} \text{(3-23)}

where,
- $\varepsilon_x$: Angular position deviation of X-axis in spindle axis C
- $\varepsilon_y$: Angular position deviation of Y-axis in spindle axis C
- $\delta_x(C)$: Positioning deviation of X-axis in spindle axis C
- $\delta_y(C)$: Positioning deviation of Y-axis in spindle axis C

\[
E_z = \begin{bmatrix}
1 & 0 & 0 & \delta_z(z) \\
0 & 1 & 0 & \delta_z(z) \\
0 & 0 & 1 & \delta_z(z) \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  \hspace{1cm} \text{(3-24)}

\[
E_x = \begin{bmatrix}
1 & 0 & 1 & \delta_z(x) \\
0 & 1 & 0 & \delta_z(x) \\
0 & 0 & 1 & \delta_z(x) \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

where,
- $\delta_z(z)$, $\delta_z(x)$: Positioning deviations
- $\delta_x(z)$, $\delta_x(x)$: Straightness deviations.

In the present research, it is assumed that the positioning errors and the straightness errors in X-axis are zero, since the motions in the X-axis are fixed in the turning processes.

### 3.5.2 Formulation of kinematic motions in turning processes

#### 3.5.2.1 Spindle motion

The spindle motion is represented by the following equation.

\[
A_{RS\_actual} = E_\theta^{-1} A_{RS}(\theta_z)^{-1} = \begin{bmatrix}
1 & 0 & -\varepsilon_y & -\delta_x(C) \\
0 & 1 & \varepsilon_x & -\delta_y(C) \\
\varepsilon_y & -\varepsilon_x & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
\cos \theta_z & \sin \theta_z & 0 & 0 \\
-\sin \theta_z & \cos \theta_z & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
3.5.2.2 Carriages motion

The motions of carriages are represented by the following equation.

\[
A_{RCR_{\text{actual}}} = [\text{Eq. (3-25)}]A^{3}_{RCR}(z)E_z
\]

\[
= \begin{bmatrix}
\cos \theta_z & \sin \theta_z & -\varepsilon_y - \delta_z(C') \\
-\sin \theta_z & \cos \theta_z & \varepsilon_y - \delta_z(C') \\
\varepsilon_y \cos \theta_z + \varepsilon_x \sin \theta_z & \varepsilon_y \sin \theta_z - \varepsilon_x \cos \theta_z & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  

(3-25)

where,

\[g_1 = \delta_y(z)\cos \theta_z - \delta_y(C) + \delta_y(z)\sin \theta_z - \varepsilon_y(\delta_v(z) + z)\]

\[g_2 = \delta_y(z)\cos \theta_z - \delta_y(C) - \delta_y(z)\sin \theta_z + \varepsilon_y(\delta_v(z) + z)\]

\[g_3 = \delta_v(z) + z + \delta_v(z)(\varepsilon_y \cos \theta_z + \varepsilon_x \sin \theta_z) - \delta_y(z)(\varepsilon_x \cos \theta_z - \varepsilon_y \sin \theta_z)\]

3.5.2.3 Cross slides motion

The motion of the cross slides are represented by the following equation based on Eq. (3-26).

\[
A_{CRCS_{\text{actual}}} = [\text{Eq. (3-26)}]A^{I}_{CRCS}(x)
\]

\[
= \begin{bmatrix}
\cos \theta_z & \sin \theta_z & -\varepsilon_y & g_1 + x \cos \theta_z \\
-\sin \theta_z & \cos \theta_z & \varepsilon_x & g_2 - x \sin \theta_z \\
\varepsilon_y \cos \theta_z + \varepsilon_x \sin \theta_z & \varepsilon_y \sin \theta_z - \varepsilon_x \cos \theta_z & 1 & g_3 + x(\varepsilon_y \cos \theta_z + \varepsilon_x \sin \theta_z) \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  

(3-26)

3.5.2.4 Tool tips motion

The motion of the tool tips are represented by the following equation based on Eq. (3-27).

\[
A_{CST_{\text{actual}}} = [\text{Eq. (3-27)}]A^{I}_{CST}(-L_x)A^{3}_{CST}(-L_z)
\]

\[
= \begin{bmatrix}
\cos \theta_z & \sin \theta_z & -\varepsilon_y & g_1 + g_4 \\
-\sin \theta_z & \cos \theta_z & \varepsilon_x & g_2 + g_5 \\
\varepsilon_y \cos \theta_z + \varepsilon_x \sin \theta_z & \varepsilon_y \sin \theta_z - \varepsilon_x \cos \theta_z & 1 & g_3 + g_6 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  

(3-28)
where,
\[ g_4 = x \cos \theta_z - L_x \cos \theta_z + \varepsilon_y L_z \]
\[ g_5 = L_x \sin \theta_z - x \sin \theta_z - \varepsilon_y L_z \]
\[ g_6 = x(\varepsilon_y \cos \theta_z + \varepsilon_x \sin \theta_z) - L_x(\varepsilon_x \cos \theta_z + \varepsilon_x \sin \theta_z) - L_z \]

3.5.3 Formulation of kinematic motions of cutting edge in turning processes

Formulation of kinematic motions of cutting edge in turning processes use formulae similar to those used in boring processes. The generalized geometric model of the inserted tool in turning processes is shown in Fig. 3-13, which are presented starting with the placement of the tool tips on the cutter bodies, the identification of oblique tool angles needed in the cutting mechanics model and the kinematics of cutting operations. The geometry of the tool tips are defined in their local coordinate system \( O_T \) analytically. The control points are derived as functions of insert parameters that have been adapted from Kaymakci et al. (2012). The coordinates of the control points are given as follows based on the standardized tool tip geometries.

\[ CRP(x, y, z) = (0, 0, 0) \]
\[ A(x, y, z) = (0, 0, -b_z + r_z (\cot \kappa_x - \csc \kappa_x)) \]
\[ B(x, y, z) = (0, 0, r_z (\cot \kappa_x - \csc \kappa_x)) \]
\[ C(x, y, z) = (r_z, 0, r_z (\cot \kappa_x - \csc \kappa_x)) \]
\[ D(x, y, z) = (r_z (1 - \csc \kappa_x), 0, r_z \cos \kappa_x \tan \frac{\kappa_x}{2}) \]
\[ E(x, y, z) = (L \sin \kappa_x - \csc \epsilon_x \sin (\epsilon_x + \kappa_x) (b_z \sin \kappa_x - (r_z \cos \kappa_x - 1)), 0, A_z + L \cos \kappa_x - \cos (\epsilon_x + \kappa_x) \csc \epsilon_x (b_z \sin \kappa_x - (r_z \cos \kappa_x - 1))) \]
\[ O_T(x, y, z) = \left( \frac{1}{2} (L \sin \kappa_x + iw \sin (\epsilon_x + \kappa_x) - 2 \csc \epsilon_x \sin (\epsilon_x + \kappa_x) (b_z \sin \kappa_x - r_z (\cos \kappa_x - 1))), 0, A_z + \frac{1}{2} (L \cos \kappa_x + iw \cos (\epsilon_x + \kappa_x) - 2 \cos (\epsilon_x + \kappa_x) \csc \epsilon_x (b_z \sin \kappa_x - r_z (\cos \kappa_x - 1))) \right) \]

(3-29)
Fig. 3-13 Tool tips and cutting edges of turning processes (Adapted from Kaymakci, et al., 2012)

The motions of the cutting edge in tool tips are represented by the following equation based on Eq. (3-29) and multiplied with Eq. (3-28).

\[
A_{TE, actual} = \left[ Eq. (3-28) \right] A_{TE}^T ( -x_E ) A_{TE}^T ( z_E )
\]

\[
x_E = \frac{1}{2} ( L \sin \kappa_r + i \omega \sin ( \varepsilon_r + \kappa_r ) - 2 \csc \varepsilon_r \sin ( \varepsilon_r + \kappa_r ) ( b_x \sin \kappa_r - r_y ( \cos \kappa_r - 1 )) )
\]

\[
z_E = A_{z} + \frac{1}{2} ( L \cos \kappa_r + i \omega \cos ( \varepsilon_r + \kappa_r ) - 2 \cos ( \varepsilon_r + \kappa_r ) \csc \varepsilon_r ( b_y \sin \kappa_r - r_x ( \cos \kappa_r - 1 )) )
\]

\[
A_{TE, actual} = \begin{bmatrix}
\cos \theta_z & \sin \theta_z & -\varepsilon_z & g_1 + g_4 & 1 & 0 & 0 & -x_E \\
-\sin \theta_z & \cos \theta_z & \varepsilon_z & g_2 + g_5 & 0 & 1 & 0 & 0 \\
\varepsilon_y \cos \theta_z + \varepsilon_z \sin \theta_z & \varepsilon_y \sin \theta_z - \varepsilon_z \cos \theta_z & 1 & g_3 + g_6 & 0 & 0 & 1 & z_E \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

(3-30)

The kinematic motions of the cutting edge of turning processes against the reference coordinate system \( O_R \) are presented by Eq. (3-31) including the kinematic deviations.

\[
P_{x, Turning} = \delta_x ( z ) \cos \theta_z - \varepsilon_x G_3 - \delta_y ( C ) - L_x \cos \theta_z + \delta_y ( z ) \sin \theta_z + x \cos \theta_z - \cos \theta_z G_2 - \varepsilon_x ( \delta_c ( z ) + z ) + \varepsilon_x L_z
\]

\[
P_{y, Turning} = \varepsilon_y G_3 - \delta_y ( C ) + \delta_y ( z ) \cos \theta_z - \delta_z ( z ) \sin \theta_z + L_x \sin \theta_z - x \sin \theta_z + \sin \theta_z G_2 + \varepsilon_y ( \delta_c ( z ) + z ) - \varepsilon_y L_z
\]

\[
P_{z, Turning} = \delta_z ( z ) - L_z - b_z + z + x G_1 - G_f G_2 + G_6 + G_7 + G_9 + \delta_z ( z ) G_1 + \delta_x ( z ) \varepsilon_x \cos \theta_z - \varepsilon_y \sin \theta_z ) - L_x G_1 - G_4
\]

(3-31)
where,
\[
G_1 = \varepsilon_1 \cos \theta_1 + \varepsilon_1 \sin \theta_1
\]
\[
G_2 = \frac{iw \sin(\varepsilon_1 + \kappa_1)}{2} + \frac{L \sin \kappa_1}{2} - \frac{\sin(\varepsilon_1 + \kappa_1) G_8}{\sin \varepsilon_1}
\]
\[
G_3 = G_6 - b_s + G_7 + G_3 - G_4
\]
\[
G_4 = \frac{\cos(\varepsilon_1 + \kappa_1) G_4}{\sin \varepsilon_1}
\]
\[
G_5 = r_s \left( \cot \kappa_1 - \frac{1}{\sin \kappa_1} \right); \quad G_6 = \frac{iw \cos(\varepsilon_1 + \kappa_1)}{2}
\]
\[
G_7 = \frac{L \cos \kappa_1}{2}; \quad G_8 = b_s \sin \kappa_1 - r_s (\cos \kappa_1 - 1)
\]

### 3.6 Virtual Machining Systems

#### 3.6.1 Virtual machining systems for boring and turning processes

The virtual machining systems have been developed to simulate the boring and turning processes, and a set of points are analytically generated to represent the bored and turned faces including the kinematic deviations. Figure 3-14 illustrates the procedure of the virtual boring and turning processes based on the input data of the kinematic deviations, the inserted tool tip parameters and the cutting conditions $f$, $v_c$, $a_p$ and $t_w$ are feed rate, cutting speeds, depth of cut and lengths of bored holes and turned parts, respectively.

The generation and estimation of the position of cutting edges with kinematic deviations against the reference coordinate system are carried out base on Eq. 3-20 for boring processes and Eq. 3-31 for turning processes. These can be transformed into the final tool position by incorporating the geometric error parameters, which are given randomly based on the normal distributions.

Once the positions of the cutting edges are obtained, the next step is to generate all points on the generated faces of the workpieces. The cutting edges of the single point cutting tool has a nose radius; therefore, the contact points of the cutting edges and the workpieces can be described in the cross sectional plane represented by the YZ-plane for boring processes and the XZ-plane for turning processes as shown in Fig. 3-15. The critical tool angles are estimated for representing the extent of the tool nose, which generate the machined face. When the kinematic deviations are considered, the tool nose centers are no longer on a straight line, and the extents of the tool nose also changes continuously. Under these conditions, the discretized tool nose radius can then be represented and evaluated.
(a) Boring processes

(b) Turning processes

Fig. 3-14 Flow chart for simulations of the boring and turning processes
In Fig. 3-15, $C_{i-1}$, $C_i$, and $C_{i+1}$ are the tool nose center locations at three consecutive revolutions of the spindle, $r_e$ is the corresponding tool nose radius, and $P_{int-1}$, $P_{int+1}$ are the intersection point and the tool contact angle $\phi_{ij-1}$ and $\phi_{ij+1}$. Based on the locations of the tool nose centers for three consecutive revolutions at the same angular position of the spindle, the tool nose can be discretized. The coordinates of the intersection point $P_{int-1}$ and $P_{int+1}$ of the tool nose profiles between the tool nose locations $C_{i-1}$, $C_i$ and $C_{i+1}$ can be calculated as follows that have been adapted from Ramaswami (2010):

In triangles $P_{int-1}P_{mid-1}C_{i-1}$ and $P_{int-1}P_{mid-1}C_i$,

\[
\begin{align*}
    a_{i-1}^2 + h_{i-1}^2 &= r_e^2; \quad h_{i-1}^2 + h_{i-1}^2 = r_e^2; \quad \text{Let } a_{i-1} + h_{i-1} = d_{i-1}; \\
    \therefore a_{i-1} &= \frac{d_{i-1}}{2}; \quad h_{i-1} = \sqrt{r_e^2 - a_{i-1}^2}; \\
    P_{mid-1} &= C_i + \frac{a_{i-1}(C_{i-1} - C_i)}{d_{i-1}}; \\
    Y_{int-1} &= Y_{mid-1} + \frac{h_{i-1}(Z_{i-1} - Z_i)}{d_{i-1}}; \\
    Z_{int-1} &= Z_{mid-1} + \frac{h_{i-1}(Y_{i-1} - Y_i)}{d_{i-1}}.
\end{align*}
\]  

(3-32)
In triangles \( P_{int+1} \), \( P_{mid+1} \), \( C_{i+1} \) and \( P_{int+1} \), \( P_{mid+1} \), \( C_{i} \) can be calculated similarly with the triangles \( P_{int-1} \), \( P_{mid-1} \), \( C_{i-1} \) and \( P_{int-1} \), \( P_{mid-1} \), \( C_{i} \).

The start and end contact angles of the tool nose with the workpiece can be calculated by the following equations that have been adapted from Ramaswami (2010):

In triangles \( P_{int+1} \), \( P_{mid+1} \), \( C_{i+1} \), Eq. (3-33) is obtained.

\[
\phi_{ij} = \cos^{-1} \left[ \frac{P_{int-i}C_{i} \cdot C_{i-1}C_{j}}{P_{int-i}C_{i}} \right]
\]  

(3-33)

In triangles \( P_{int+1} \), \( P_{mid+1} \), \( C_{i+1} \), Eq. (3-34) is obtained.

\[
\phi_{ij+1} = \cos^{-1} \left[ \frac{P_{int+i}C_{i} \cdot C_{i+1}C_{j}}{P_{int+i}C_{i}} \right]
\]  

(3-34)

At any given tool nose locations related to the angular orientation ‘\( \theta ' \) of the spindle with respect to the reference orientation, the span of the tool contact angle between the start and end angle can be divided as required. The profile of the machined face can then be generated as a function of the final position of the cutting edges considering all kinematic deviations \( x''' \), \( y''' \), \( z''' \), where, \( x''' \), \( y''' \), \( z''' \) are the geometric deviations of position in the X, Y and Z-axis, obtained from the kinematic motion deviations of both the spindle and the Z-axis feed motions. The tool nose radius \( (r_e) \) and the tool contact angle \( (\phi_{ij+1}) \) by applying the following equation.

\[
F_i = (f) \cdot \left( \frac{\theta_z}{2\pi} \right); \quad R_{inst} = \sqrt{x'^2 + y'^2}
\]

\[
X_{surf\_boring} = (R_d + R_{inst} + r_e \cdot \cos \phi_{ij+1}) \cdot \sin \theta_z
\]

\[
Y_{surf\_boring} = (R_d + R_{inst} + r_e \cdot \cos \phi_{ij+1}) \cdot \cos \theta_z
\]

\[
Z_{surf\_boring} = F_i + z''' + r_e \cdot \cos \phi_{ij+1}
\]

(3-35)

\[
X_{surf\_turning} = (R_d + R_{inst} + r_e \cdot \sec \phi_{ij+1}) \cdot \sin \theta_z
\]

\[
Y_{surf\_turning} = (R_d + R_{inst} + r_e \cdot \sec \phi_{ij+1}) \cdot \cos \theta_z
\]

\[
Z_{surf\_turning} = F_i + z''' + r_e \cdot \cos \phi_{ij+1}
\]

(3-36)

where,

\[
X_{surf\_boring}, Y_{surf\_boring}, Z_{surf\_boring} : \text{Positions of a point on the bored faces}
\]

\[
X_{surf\_turning}, Y_{surf\_turning}, Z_{surf\_turning} : \text{Positions of a point on the turned faces}
\]
By repeating this process at each location of the tool, the entire surface of the machined part can be created. Next section illustrates sample bored holes and turned parts generated using the virtual machining module.

### 3.6.2 Simulation of 3-dimensional bored and turned faces

This section shows an example of the generated faces by boring and turning process simulations considering kinematic deviations. As shown in the figure, a set of the points on the bored and turned faces are generated. The geometric deviations of any points on the bored and turned faces can be evaluated by comparing the generated faces with the kinematic deviations. In order to validate the virtual machining module of boring and turning processes, two parts are considered between the bored and turned faces without kinematic deviations and another one with kinematic deviations.

#### 3.6.2.1 Simulation of bored and turned faces without kinematic deviations

Figures 3-16 and 3-17 show the simulation of the virtual machining of the bored and turned faces without kinematic deviations, which is simulated by program MATLAB in order to show the discrepancy of the machined and nominal surfaces. The simulations of the boring and turning processes are carried out based on Eq. 3-35 and 3-36, respectively. The machining parameters of boring and turning processes consists the input data are described as follows:

1. **Machining parameters for boring processes,**
   
   \[ f = 0.25 \text{ mm/rev}, \ z = 300 \text{ mm}, \ z_B = 73 \text{ mm}, \ R_T = 10 \text{ mm}, \ r_e = 0.8 \text{ mm}; \ \kappa_r = 100^\circ, \ \epsilon_r = 80^\circ, \ iw = 6.35 \text{ mm}, \ b_s = 0 \text{ mm}, \ L = 6.35 \text{ mm}. \]

2. **Machining parameters for turning processes,**
   
   \[ f = 0.25 \text{ mm/rev}, \ x = 300 \text{ mm}, \ z = 350 \text{ mm}, \ L_x = 100 \text{ mm}, \ L_z = 150 \text{ mm}, \ R_d = 10 \text{ mm}, \ r_e = 0.8 \text{ mm}, \ \kappa_r = 100^\circ, \ \epsilon_r = 80^\circ, \ iw = 6.35 \text{ mm}, \ b_s = 0 \text{ mm}, \ L = 6.35 \text{ mm}. \]

3. **All the kinematic motion deviations are set to be zero.**

The bored and turned faces are obtained from 8,000 points generated by the boring and turning process simulations in the Figs. 3-16 and 3-17. The generated surfaces are very smooth and only have the distances due to the nose radius in the tool geometries and the feed rates in the machining parameters.
Fig. 3-16 Estimated geometries of bored faces without kinematic motion deviations

Fig. 3-17 Estimated geometries of turned faces without kinematic motion deviations
3.6.2.2 Simulation of bored and turned faces with kinematic deviations

Figures 3-18 and 3-19 show the simulation of the virtual machining of the bored and turned faces with kinematic deviations which is the simulation method similar to the method of Figs. 3-16 and 3-17, but the input data of geometric deviation errors are difference. The machining parameters of boring and turning processes consists the input data are described as follows:

Figures 3-18 and 3-19 show the simulation of the virtual machining of the bored and turned faces with kinematic deviations. The machining parameters are same as ones for the simulations without kinematic motion deviations shown in the previous section.

The kinematic motion deviations are set as shown in the followings.

1. Position deviations of spindles $\delta_x(C)$ and $\delta_y(C)$ in Eqs. 3-13 and 3-23 are given randomly based on the normal distribution $N(0, 1 \mu m)$ along the spindle rotation $\theta_z$,

2. Rotational deviations of spindles $\varepsilon_x$ and $\varepsilon_y$ in Eqs. 3-13 and 3-23 are given randomly based on the normal distribution $N(0, 3 \mu rad)$ along the spindle rotation $\theta_z$,

3. Positioning deviations in Z-axis $\delta_z(z)$ in Eqs. 3-14 and 3-24 are given randomly based on the normal distribution $N(0, 5 \mu m)$ along the Z-axis feed motion $z$,

4. Straightness deviations in Z-axis $\delta_x(z)$ and $\delta_y(z)$ in Eqs. 3-14 and 3-24 are given randomly based on the normal distribution $N(0, 1 \mu m)$ along the Z-axis feed motion $z$, and

5. Positioning and straightness deviations in X-axis in Eqs. 3-14 and 3-24 are set to be 0, since the X-axis are fixed in both the boring and the turning processes.

Fig. 3-18 Estimated geometries of bored faces with kinematic motion deviations
Fig. 3-19 Estimated geometries of bored faces with kinematic motion deviations

Figures 3-18 and 3-19 show the examples of the machined surfaces obtained from 8,000 points generated by the boring and turning process simulations. The generated surfaces have larger disturbances compared with the ones shown in Figs. 3-16 and 3-17. The disturbances are due to the kinematic motion deviations of the spindle and the Z-axis table.

3.7 Conclusion

The research discussed in this chapter deals with the virtual machining of boring and turning processes including kinematic motion deviations. A simulation model for the boring and turning processes is proposed to evaluate the geometric deviations of the generated faces. The followings are concluded.

(1) A model is proposed to represent the kinematic motions of the cutting edges against the workpieces fixed on the spindles, taking into consideration of the kinematic deviations of the boring tool systems of the machining centers and turning tool systems of the turning centers. The models proposed here represent the positions of the cutting edges against the workpieces by applying the 4 by 4 transformation matrices including the kinematic motion deviations.

(2) A systematic method is proposed to estimate the geometric deviations of the machined face based on the kinematic motions of the cutting edges. A set of points on the bored and the turned faces are generated by applying the proposed method and the characteristics features of the generated faces are estimated based on the points.
(3) A proposed model and method are applied to the simulation of the simple boring and turning processes. The geometries of the bored and turned faces are estimated, based on the cutting conditions, the tool geometries and the kinematic deviations of the boring and turning processes.
Chapter 4

Estimation of 3-Dimensional Surface Roughness Including Kinematic Motion Deviations
4. Estimation of 3-Dimensional Surface Roughness Including Kinematic Motion Deviations

4.1 Introduction

In the present global competitive situation, manufacturing processes in industry are developing continuously to increase the quality and the reliability of their products and to decrease the cost of the manufacturing process operations. In general it is problematic to determine the values of the process parameters in practice and to maximize the manufacturing system performance using the available resources.

The surface finish of the machined parts is one of the most important product quality characteristics. Surface roughness greatly influences the mechanical and physical properties of the contacting parts. The understanding of the behavior is important in many applications such as wear, friction, lubrication, sealing tightness of joint, contact rigidity, contact stress, loaded area and thermal conductivity. Therefore, engineers are very concerned about the surface roughness of the products (Dong, et al., 1992).

![Diagram](image)

**Fig. 4-1 Key components of estimation of the 3D surface roughness**

The development of new industries has led to a requirement for super-smooth surfaces and for the ability to measure and predict surfaces roughness of industrial parts before real machining processes. The estimation of 3-dimensional (3D) surface roughness of machined parts before real machining process is important for the design and the manufacturing process of products. The
simulation of virtual machining is one method of estimating the 3D surface roughness from virtualize parts of machining processes. This section discusses the 3D surface roughness that can be estimated for the virtual part profiles as shown in Fig. 4-1.

The key components of estimation of 3D surface roughness based on simulation of virtual machining module in this chapter are:

- Reference datum for cylinder assessment
- Mathematical models of least mean squares
- Formulation for estimation of the surface roughness
- Case Study

Details of these models are provided below:

**4.2 Reference Datum for Cylinder Assessment**

In recent years, the assessment of 3D surface roughness has become more and more popular in both academia and industries. However it would seem that the choice of suitable reference planes is still discretionary and arbitrary. Although some reference planes have already been extended from reference lines and applied to 3D topography characterization, little information is available on the relative efficiency and feasibility of the reference planes, especially for those used for nominally curved surfaces. From a standardization point of view this causes two problems. The first is that it is difficult for users to determine what reference planes to use and the second is that it is difficult to carry out a comparison between measurement results which are obtained by using different reference planes (Dong, et al., 1995). In order to be able to find a suitable reference datum for 3D topography characterization, this section starts with assessment cylinder of the measurement of form deviations.

Measurements of form deviations of engineering components are carried out with respect to a reference datum or trajectory. For example, straightness measurements are carried out with reference to a straight edge and roundness measurements with reference to a circular trajectory. The deviations have to be measured against the nominal geometric elements. ISO specifies that the actual measurement such that maximum deviation between the datum and the actual features concerned is the least mean squares for evaluating the form errors (Shunmugam, 1987). The assessment cylinders shown in Fig. 4-2 are considered here as the reference datum to measure the 3D surface roughness of bored and turned surfaces. The assessment cylinders are generated based on all the 3D coordinate data generated by the 3D boring and turning process simulations.
A procedure is proposed to generate an assessment cylinder in order to verify the 3D surface roughness based on all the trajectories of the cutting edge in the boring and turning processes. The parameters to be determined here are summarized in the following, which represent a cylinder in the reference coordinate system $X_RY_RZ_R$. 

Fig. 4-2 Coordinate system for measuring deviations
1. Unit vector \((l_0, m_0, n_0)\) representing the direction of the axis of the assessment cylinder,
2. Vector \((a_0, b_0, 0)\) representing the position of the axis of the assessment cylinder,
3. \(R_0\) representing the radius of the assessment cylinder,
4. \(E_i\) representing the deviation of \(i\)th point respect to the least mean square cylinders,
5. \(X_i, Y_i, Z_i\) representing the Cartesian coordinates of \(i\)th point,
6. \(r_i\) representing the radius vector of Cartesian coordinate of \(i\)th point, given as \(r_i = \sqrt{X_i^2 + Y_i^2}\),
7. \(\theta_i\) representing the angular coordinate of Cartesian coordinate of \(i\)th point,
8. \(e_i\) representing the eccentricity given as \(e_i = \sqrt{a_0^2 + b_0^2}\),
9. \(\alpha, \beta\) representing the angular coordinate of X-axis and Y-axis, respectively,
10. \(\phi, \psi\) representing the rotational angle around X-axis and Y-axis, respectively.

4.3 Mathematical Models of Least Mean Squares

4.3.1 Derivation of least mean squares (Thomas and Chan, 1989)

The reference coordinate system \(X_RY_RZ_R\) coincides the one in Fig. 3-7 for boring processes and Fig. 3-8 for turning processes, therefore, the parameters \(l_0\) and \(m_0\) are fixed, the parameters \(a_0, b_0,\) and \(R_0\) are estimated by applying the following equations, based on the least mean square method proposed by Thomas and Chan (1989).

Given a set of coordinate \((X_i, Y_i)\)…\((X_i, Y_i)\)…\((X_N, Y_N)\), define a circle with center \((a_0, b_0)\) and radius \(R_0\). We define the error as the difference between the constant area \(\pi R^2\) and the area of the circle centered at \(a_0, b_0\) and has radius

\[ (X_i - a_0)^2 + (Y_i - b_0)^2 \]^{1/2}.

Summing up the squares of the errors we have

\[ f(R_0, a_0, b_0) = \sum_{i=1}^{N} \left[ \pi R_0^2 - \pi \left( (X_i - a_0)^2 + (Y_i - b_0)^2 \right) \right]^2 \]

or

\[ J = \frac{f}{\pi^2} = \sum_{i=1}^{N} \left[ R_0^2 - \left( (X_i - a_0)^2 + (Y_i - b_0)^2 \right) \right]^2. \] (4-1)

The function \(J(R_0, a_0, b_0)\) should be minimized with respect to \(R_0, a_0\) and \(b_0\). Differentiating Eq. (4-1) with respect to \(R_0\) yields
\[
\frac{\partial J}{\partial R_0} = 2 \sum_{i=1}^{N} \left[ R_0^2 - \left\{ X_i - a_0 \right\}^2 + (Y_i - b_0)^2 \right] R_0 = 0
\]

or

\[
NR_0^2 = \sum_{i=1}^{N} \left\{ X_i - a_0 \right\}^2 + (Y_i - b_0)^2 \] (4-2)

Differentiating Eq. (4-1) with respect to \( a_0 \) yields

\[
\frac{\partial J}{\partial a_0} = 2 \sum_{i=1}^{N} \left[ R_0^2 - \left\{ X_i - a_0 \right\}^2 + (Y_i - b_0)^2 \right] X_i = 0
\]

or

\[
\sum_{i=1}^{N} \left[ R_0^2 - \left\{ X_i - a_0 \right\}^2 + (Y_i - b_0)^2 \right] X_i = \sum_{i=1}^{N} \left[ R_0^2 - \left\{ X_i - a_0 \right\}^2 + (Y_i - b_0)^2 \right] R_0 = 0
\]

(since RHS (right hand side) is zero from Eq. (4-2). Therefore,

\[
R_0^2 \sum_{i=1}^{N} X_i = \sum_{i=1}^{N} \left\{ X_i - a_0 \right\}^2 + (Y_i - b_0)^2 X_i. \] (4-3)

Similarly differentiating Eq. (4-1) with respect to \( b_0 \) yields

\[
R_0^2 \sum_{i=1}^{N} Y_i = \sum_{i=1}^{N} \left\{ X_i - a_0 \right\}^2 + (Y_i - b_0)^2 Y_i. \] (4-4)

Equations (4-2), Eq. (4-3) and Eq. (4-4) can be solved, even though quadratic, using the following tricks. First, let us simplify the notations using the following conventions. We use summation over \( n \) indices as \( \sum_{i} \) and let \( \Sigma \) stand for \( \sum_{i} X_i = X_1 + X_2 + \ldots + X_i + \ldots + X_N \) and

\[
\Sigma Y = \sum Y_i \quad \Sigma X^2 = \sum X_i^2 \quad \Sigma Y^2 = \sum Y_i^2 \]

\[
\Sigma XY = \sum X_i Y_i \quad \Sigma X^3 = \sum X_i^3 \quad \Sigma Y^3 = \sum Y_i^3 \]

\[
\Sigma X^2 Y = \sum X_i Y_i \quad \Sigma XY^2 = \sum X_i Y_i^2 \]

With these conventions, Eq. (4-2) becomes

\[
- 64 -
\]
\[ NR_0^2 = \sum x^2 - 2\sum x a_0 + N a_0^2 + \sum y^2 - 2\sum y b_0 + N b_0 \]  
(4-5)

and Eq. (4-3) becomes

\[ R_0^2 \sum x = \sum x^3 - 2\sum x^2 a_0 + \sum x a_0^2 + \sum x_y^2 - 2\sum x_y b_0 + \sum x b_0^2 \]  
(4-6)

and Eq. (4-4) becomes

\[ R_0^2 \sum y = \sum x^2 y - 2\sum x y a_0 + \sum y a_0^2 + \sum y_y^2 - 2\sum y_y b_0 + \sum y b_0^2. \]  
(4-7)

Equations (4-5), (4-6) and (4-7) can be solved.

Multiply Eq. (4-5) by \( \sum x \) and subtract \( N \) multiplied by Eq. (4-6) to give

\[ \sum x^2 \sum x - N \sum x^3 - 2a_0 \left[ \sum x^2 - N \sum x^2 \right] + \sum x \sum y^2 - N \sum x y^2 - 2b_0 \left[ \sum x \sum y - N \sum x y \right] = 0 \]  
(4-8)

Multiply Eq. (4-5) by \( \sum y \) and subtract \( N \) multiplied by Eq. (4-7) to give

\[ \sum x^2 \sum y - N \sum x^2 y - 2a_0 \left[ \sum x \sum y - N \sum x y \right] + \sum y \sum y^2 - N \sum y y^2 - 2b_0 \left[ \sum y^2 - N \sum y y \right] = 0 \]  
(4-9)

We can now solve Eq. (4-8) and (4-9) for \( a_0 \) and \( b_0 \); the result can be written in matrix form as

\[
\begin{bmatrix}
    a_1 \\
    a_2
\end{bmatrix}
\begin{bmatrix}
    a_0 \\
    b_0
\end{bmatrix}
=
\begin{bmatrix}
    c_1 \\
    c_2
\end{bmatrix}
\]  
(4-10)

where,

\[ a_1 = 2 \left[ \sum x^2 - N \sum x^2 \right] \]
\[ b_1 = 2 \left( \sum x \sum y - N \sum x y \right) \]
\[ a_2 = 2 \left( \sum x \sum y - N \sum x y \right) = b_1, \]
\[ b_2 = 2 \left[ \sum y^2 - N \sum y y \right] \]
\[ c_1 = \left( \sum x^2 \sum x - N \sum x^3 + \sum x \sum y^2 - N \sum x y^2 \right), \]
\[ c_2 = \left( \sum x^2 \sum y - N \sum y^3 + \sum y \sum y^2 - N \sum x y^2 \right). \]
from which

$$a_0 = \frac{c_1b_2 - c_2b_1}{a_1b_2 - a_2b_1}$$  \hspace{1cm} (4-11)

and

$$b_0 = \frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1}.$$  \hspace{1cm} (4-12)

Having solved \(a_0, b_0\) we can substitute into Eq. (4-5) to get \(R_0^2\), where

$$R_0^2 = \frac{1}{N} \left\{ \sum_{i=1}^{N} x_i^2 - 2\sum_{i=1}^{N} x_i a_0 + Na_0^2 + \sum_{i=1}^{N} y_i^2 - 2\sum_{i=1}^{N} y_i b_0 + Nb_0^2 \right\}$$  \hspace{1cm} (4-13)

The least mean square errors \(E_i\) of all the generated point \(P_i(X_i, Y_i, Z_i)\) and the assessment cylinder is given in the following equation, as shown in Fig. 4-2. Therefore, the transformation matrices between the reference coordinate system \((X_R, Y_R, Z_R)\) and the local coordinate for cylinder \((xyz)\) are given as follows.

$$\begin{bmatrix}
 c_\psi & 0 & -s_\psi & 0 \\
 0 & 1 & 0 & 0 \\
 s_\psi & 0 & c_\psi & 0 \\
 0 & 0 & 0 & 1
\end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\
 0 & 1 & 0 & 0 \\
 0 & 0 & 1 & 0 \\
 0 & 0 & 0 & 1
\end{bmatrix} = \begin{bmatrix} X_R \\
 Y_R \\
 Z_R \\
 1
\end{bmatrix}$$

\begin{align*}
x &= X_R \cos \phi + Y_R \sin \phi \sin \psi - Z_R \sin \phi \cos \phi - a_0 \cos \phi - b_0 \sin \phi \sin \phi \\
y &= Y_R \cos \phi + Z_R \sin \phi - b_0 \cos \phi \\
z &= X_R \sin \phi - Y_R \cos \phi \sin \phi + Z_R \cos \phi \cos \phi - a_0 \sin \phi + b_0 \cos \phi \sin \phi
\end{align*}

(4-14)

Assume \(\psi = 0\) and \(\phi = 0\), therefore, Eq. (4-15) is obtained.

$$\begin{align*}
x &= X_R - \psi Z_R - a_0 \\
y &= Y_R + \phi Z_R - b_0 \\
z &= Z_R + \psi(X_R - a_0) - \phi(Y_R - b_0)
\end{align*}$$

(4-15)
Given $l_0 = \psi$ and $m_0 = -\phi$, therefore, the Eq. (4-15) is used in the least mean square errors $E_i$ of all the generated point $P_i(X_i, Y_i, Z_i)$ and the assessment cylinder is given in the following equation.

$$E_i = \left( (X_i - l_0Z_i - a_0)^2 + (Y_i - m_0Z_i - b_0)^2 \right)^{1/2} - R_0$$  \hspace{1cm} (4-16)

The sum of the squares of the deviation is as follows.

$$E_s = \sum_{i=1}^{N} E_i^2$$  \hspace{1cm} (4-17)

### 4.3.2 Algorithm for calculating parameters

The procedure for calculating parameters is shown in Fig. 4-3, all data points $P_i(X_i, Y_i, Z_i)$ are generated by the 3D boring and turning process simulations. The algorithm for calculating parameters $a_0$, $b_0$, $l_0$, $m_0$ and $R_0$ of the assessment cylinders are estimated by applying the following procedure.

**STEP 1: Setting of parameters $l_0$ and $m_0$**

The parameters $l_0$ and $m_0$ are set in the initial values -0.010 and moved to +0.010 by the step of 0.001. These parameters are input into by applying the following matrices.

$$\begin{bmatrix} x \\ y \\ z \\ l \end{bmatrix} = \begin{bmatrix} 1 & 0 & -m & 0 & X_R \\ 0 & 1 & -l & 0 & Y_R \\ m & l & 1 & 0 & Z_R \\ 0 & 0 & 0 & l & I \end{bmatrix}$$  \hspace{1cm} (4-18)

**STEP 2: Estimation of parameters $a_0$, $b_0$, and $R_0$**

The parameters $a_0$, $b_0$, and $R_0$ are estimated by applying the Eqs. (4-11) to (4-13) for all the combination of $l_0$ and $m_0$.

**STEP 3: Estimation of least mean square errors $E_i$ and select an optimal parameters.**

The least mean square errors $E_s$ is calculated by applying Eq. (4-17) based on the parameters of $a_0$, $b_0$, $l_0$, $m_0$, $R_0$ and the generated points $P_i(X_i, Y_i, Z_i)$. A combination of the parameter values which minimize $E_s$ is selected as the parameters representing the assessment cylinder. The parameters $a_0$, $b_0$, $l_0$, $m_0$ and $R_0$ obtained here give the position and orientation errors and the dimensional error in the radius of the bored and turned cylinders generated by the simulations.
4.4 Formulation for Estimation of the Surface Roughness

4.4.1 Two-dimensional surface roughness

Surfaces are characterized by three main parameters: roughness, waviness, and lay. Roughness means the irregularities of a surface, usually resulting from a production process or usage, such as machining or wear. These irregularities are typically submicron in scale. Waviness describes a more widely spaced component on the surface, with larger dimensions; for instance, the deviation from flatness. So, thinking analogously, waviness is a kind of carrier wave and roughness is the modulation over it. Lay is defined as the main direction of the surface texture. This is often determined by the production method, such as the direction of grinding, turning etc. These surface characteristics are depicted schematically in Fig. 4-4 (Rantala, 2004).

Roughness is measured by contact profilometer-type devices that ride on the surface on a skid—which provides the reference for the measurements or non-contact devices such as interferometry-based or, recently, atomic force microscopes. The instrument used is dependent on the resolution needed for the measurement. This research does not discuss these instruments because the research focuses on estimated simulation of virtual machining.
After generating the assessment cylinders, surface roughness is estimated as the theoretical surface roughness of maximum height ($R_{z_{th}}$), the theoretical surface roughness of arithmetical mean deviation ($R_{a_{th}}$), the 2-dimensional (2D) surface roughness of maximum height of the assessed profile ($R_z$), arithmetical mean deviation of the assessed profile ($Ra$) and root mean square of the assessed profile ($Rq$) defined in ISO 4287:1997 and JIS B 0601:2001 are estimated by Eqs. (4-19) to (4-23). The formulations of the 2D surface roughness are estimated by applying the following.

4.4.1.1 Theoretical surface roughness

The idealized model for the surface roughness using a tool having a nose radius ($r_e$) used in boring and turning processes with feed rate ($f$) is shown in Fig. 4-5. $R_{z_{th}}$, the surface roughness defined in JIS B 0601:2001, can be estimated by the surface roughness show in Eq. (4-19), and $R_{a_{th}}$ defined by Dornfeld and Lee (2010) is shown in Eq. (4-20).

$$ R_{z_{th}} = \frac{f^2}{8r_e} $$ (4-19)
4.4.1.2 Maximum height of the assessed profile

The 2D surface roughness of $R_z$ is obtained from the distance between the highest peak and the lowest valley in the range of sampled reference length ($\ell$) in the direction of mean line of the roughness curve. After generating the assessment cylinders, the surface roughness parameter, $R_z$, can be estimated according to JIS B 0601:2001 by measuring the lines of 12.5 mm according to JIS B 0633:2001, as shown by the formula in Eq. (4-21).

$$R_z = R_p + R_v = \max(E_i) + \min(E_i)$$  \hspace{1cm} (4-21)

4.4.1.3 Arithmetical mean deviation of the assessed profile

The 2D surface roughness of $Ra$ defined in JIS B 0601:2001 is obtained from the following formula when the roughness curve is expressed by $y = f(x)$, taking X-axis to the mean line direction and Y-axis to the vertical magnification of the roughness curve in the range of sampled reference length “$\ell$”. An indicative calculation of $Ra$ is shown in Fig. 4-7, 4-8 and Eq. (4-22).
The 2D surface roughness of $Rq$ defined in ISO 4287:1997 is obtained from the following formula when the roughness curve is expressed by $y = f(x)$, taking X-axis to the mean line direction and Y-axis to the vertical magnification of the roughness curve in the range of sampled reference length “$\ell$”. An indicative calculation of $Rq$ is shown in Fig. 4-8 and Eq. (4-23).

$$
Rq = \left(\frac{1}{\ell} \int_0^\ell y^2 \, dx\right)^{1/2} \equiv \left(\frac{y_a^2 + y_b^2 + y_c^2 + \ldots + y_N^2}{N}\right)^{1/2} = \left(\frac{1}{N} \sum_{i=1}^{N} |E_i|\right)^{1/2}
$$

(4-23)
4.4.2 Three-dimensional surface roughness (Blateyron, 2013)

Continuing from generating the assessment cylinders and estimating the 2D surface roughness, the 3D surface roughness which has not yet been addressed widely, should be considered. In the following, a capital letter “S” (for Surface) is used to identify 3D parameters, in order to distinguish 2D and 3D parameters according to ISO 25178 part 2.

The 3D surface roughness was proposed by Dong et al. (1994), Quinsat et al. (2008) and Abouelatta (2010) to assess the 3D surface topography based on the least mean squares plans. The parameters of maximum height of the surface ($S_z$), in this research has been adapted from Blateyron (2013), and the arithmetic mean height of the surface ($S_a$) and root mean square height of the surface ($S_q$) have been adapted from Abouelata (2010), Dong et al. (1994), Quinsat et al. (2008). The 3D surface roughness parameters are estimated by applying the following equations.

4.4.2.1 Sampling area

Profile parameters are defined based either on a sampling length or the evaluation length. If a parameter is defined on a sampling length, it is (by default) calculated on each sampling length (ISO 4288:1996) and a mean value calculated (the default number of sampling lengths is five). With surfaces and areal parameters, the concepts of sampling and evaluation areas are still defined but the default is one sampling area per evaluation area. This simply means that parameters are calculated on the measured surface without segmenting the surface into small sub-areas that depend on the sampling length (Blateyron, 2013).

4.4.2.2 Maximum height of the Surface

The 3D surface roughness parameter of maximum height of the surface ($S_z$) has been similarly estimated with the 2D surface roughness that is evaluated by measuring the lines of 12.5 mm, but 3D surface roughness is evaluated by measuring the areas 12.5 mm x 12.5 mm square from the points of the generated faces referring to JIS B 0633:2001.

Figure 4-9, the $S_p$ parameter represents the maximum peak height, that is to say the height of the highest point of the surface. The $S_v$ parameter represents the maximum pit height, i.e. the height of the lowest point of the surface. As heights are counted from the mean plane and are signed, $S_p$ is always positive and $S_v$ is always negative (Blateyron, 2013). Equation (4-24) is sum of the maximum height of the surface by applying the following equations.

$$S_z = S_p + |S_v| = max\{E_i\} + min\{|E_i|\}$$

(4-24)
4.4.2.3 Arithmetic mean and root mean square height of the surface

The 3D surface roughness parameter of the arithmetic mean of the surface ($S_a$) and root mean square height of the surface ($S_q$) are defined as the arithmetic mean of the absolute value of the height within a sampling area $A$ and is expressed by $z(x, y)$ (Blateyron, 2013). The parameter is evaluated by measuring the areas 12.5 mm x 12.5 mm square from the points of the generated faces referring to JIS B 0633:2001 by applying the following equations.

$$S_a = \frac{1}{A} \iint_{A} |z(x, y)| \, dx \, dy = \frac{1}{N} \sum_{i=1}^{N} |E_i|$$

$$S_q = \left( \frac{1}{A} \iint_{A} z^2(x, y) \, dx \, dy \right)^{1/2} \equiv \left( \frac{1}{N} \sum_{i=1}^{N} (E_i)^2 \right)^{1/2}$$

4.5 Case Study

This section shows an example of faces generated by boring and turning process simulations considering kinematic deviations. As shown in Figs. 3-16 to 3-19, a set of points on the bored and turned faces are generated. The geometric deviations of any points on the bored and turned faces can be evaluated by comparing the generated faces with the kinematic deviations. The surface roughness of the generated faces are summarized in Tables 4-1, 4-2 and 4-3 based on the simulation results under twelve different machining parameters of the feed rates and tool nose radius of the inserted tool tips. The 2D and 3D surface roughness is evaluated by measuring the lines of 12.5 mm length and the areas 12.5 mm x 12.5 mm square referring to JIS B 0633:2001.
In order to validate the virtual machining module of boring and turning processes, two parts are considered between the bored and turned faces without kinematic deviations and another one with kinematic deviations.

### 4.5.1 Estimation of bored and turned faces without kinematic deviations

Table 4-1 shows the comparisons between the theoretical values of the 2D surface roughness calculated from the feed rates and the nose radius, and the 2D and the 3D surface roughness obtained through the simulations shown in Figs. 3-16 and 3-17. As regards the 2D surface roughness, the estimated values of $Ra$ and $Rz$ are almost the same as the theoretical values $Ra_{th}$ and $Rz_{th}$. The estimated values are obtained from 20,000 points generated by the boring and turning process simulations. The percentage of maximum differences between the results of the theoretical and estimated values is less than 5.84 % for $Ra$ and 7.24 % for $Rz$.

Table 4-1 Estimated 2D and 3D surface roughness of boring and turning process simulations without kinematic deviations

<table>
<thead>
<tr>
<th>Test no.</th>
<th>Cutting Condition</th>
<th>Surface Roughness without Kinematic Motion Deviations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Nominal Diameter</td>
<td>$Rz_{th}$ [µm]</td>
</tr>
<tr>
<td></td>
<td>$f$ [mm/rev]</td>
<td>$r_e$ [mm]</td>
</tr>
<tr>
<td>1</td>
<td>40.0</td>
<td>0.05</td>
</tr>
<tr>
<td>2</td>
<td>40.0</td>
<td>0.1</td>
</tr>
<tr>
<td>3</td>
<td>40.0</td>
<td>0.2</td>
</tr>
<tr>
<td>4</td>
<td>40.0</td>
<td>0.05</td>
</tr>
<tr>
<td>5</td>
<td>40.0</td>
<td>0.1</td>
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<tr>
<td>6</td>
<td>40.0</td>
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<tr>
<td>7</td>
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<tr>
<td>11</td>
<td>40.0</td>
<td>0.1</td>
</tr>
<tr>
<td>12</td>
<td>40.0</td>
<td>0.2</td>
</tr>
</tbody>
</table>
4.5.2 Estimation of bored and turned faces with kinematic deviations

This section shows the estimation of 2D and 3D surface roughness based on simulation of the virtual machining of the bored and turned faces with kinematic deviations. The machining parameters are same as those used for the simulations with kinematic motion deviations shown in the previous chapter.

Table 4-2 Estimated 2D and 3D surface roughness of boring process simulations with kinematic deviations

<table>
<thead>
<tr>
<th>Test no.</th>
<th>Cutting Condition</th>
<th>Surface Roughness with Kinematic Motion Deviations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>40.0</td>
<td>0.05</td>
</tr>
<tr>
<td>2</td>
<td>40.0</td>
<td>0.1</td>
</tr>
<tr>
<td>3</td>
<td>40.0</td>
<td>0.2</td>
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<td>4</td>
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<td>0.05</td>
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<td>0.05</td>
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<td>0.2</td>
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<tr>
<td>12</td>
<td>40.0</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Tables 4-2 and 4-3 summarize the 2D surface roughness $R_{z\_th}$, $R_{a\_th}$, $R_z$, $R_a$ and $R_q$, the 3D surface roughness $S_z$, $S_a$ and $S_q$ obtained from the boring and turning process simulations under the various conditions of the feed rate $f$ and the nose radius of the cutting edges $r_e$. The obtained surface roughness is much greater than the theoretical values and also the simulation results shown in Table 4-1 due to the kinematic motion deviations. The evaluated surface roughness is not so affected by the feed rate $f$ and the nose radius $r_e$ as the case of the simulation without kinematic motion deviations. This means that effects of the kinematic motion deviations are...
greater than the effects of the machining parameters. The estimated values are obtained from 20,000 points generated by the boring and turning process simulations.

Table 4-3 Estimated 2D and 3D surface roughness of turning process simulations with kinematic deviations

<table>
<thead>
<tr>
<th>Test no.</th>
<th>Nominal Diameter [mm]</th>
<th>Cutting Condition</th>
<th>Surface Roughness with Kinematic Motion Deviations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Theoretical Simulations</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Rz_th [µm]</td>
</tr>
<tr>
<td>1</td>
<td>40.0</td>
<td>0.05</td>
<td>0.4</td>
</tr>
<tr>
<td>2</td>
<td>40.0</td>
<td>0.1</td>
<td>0.4</td>
</tr>
<tr>
<td>3</td>
<td>40.0</td>
<td>0.2</td>
<td>0.4</td>
</tr>
<tr>
<td>4</td>
<td>40.0</td>
<td>0.05</td>
<td>0.8</td>
</tr>
<tr>
<td>5</td>
<td>40.0</td>
<td>0.1</td>
<td>0.8</td>
</tr>
<tr>
<td>6</td>
<td>40.0</td>
<td>0.2</td>
<td>0.8</td>
</tr>
<tr>
<td>7</td>
<td>40.0</td>
<td>0.05</td>
<td>1.2</td>
</tr>
<tr>
<td>8</td>
<td>40.0</td>
<td>0.1</td>
<td>1.2</td>
</tr>
<tr>
<td>9</td>
<td>40.0</td>
<td>0.2</td>
<td>1.2</td>
</tr>
<tr>
<td>10</td>
<td>40.0</td>
<td>0.05</td>
<td>1.6</td>
</tr>
<tr>
<td>11</td>
<td>40.0</td>
<td>0.1</td>
<td>1.6</td>
</tr>
<tr>
<td>12</td>
<td>40.0</td>
<td>0.2</td>
<td>1.6</td>
</tr>
</tbody>
</table>

The values of 3D surface roughness of Sz generally give larger values than 2D surface roughness in most cases by the machining conditions of \( f = 0.1 \text{ mm/rev} \) and \( r_e = 0.8 \text{ mm} \) for boring processes, and of \( f = 0.05 \text{ mm/rev} \) and \( r_e = 1.2 \text{ mm} \) for turning processes. The Sz and Sq are the smallest value of the surface roughness for boring processes, as obtained by the machining conditions of \( f = 0.1 \text{ mm/rev} \) and \( r_e = 1.6 \text{ mm} \), and for turning processes as obtained by the machining conditions of \( f = 0.1 \text{ mm/rev} \) and \( r_e = 1.2 \text{ mm} \). The kinematic motion deviations are set as shown below.

1. Position deviations of spindles \( \delta_x(C) \) and \( \delta_y(C) \) in Eq. (3-13) and (3-23) are given randomly based on the normal distribution \( N(0, 1 \text{ µm}) \) along the spindle rotation \( \theta_z \).
2. Rotational deviations of spindles \( \varepsilon_x \) and \( \varepsilon_y \) in Eq. (3-13) and (3-23) are given randomly based on the normal distribution \( N(0, 3 \text{ µrad}) \) along the spindle rotation \( \theta_z \).
(3) Positioning deviations in Z-axis $\delta_z(z)$ in Eq. (3-14) and (3-24) are given randomly based on the normal distribution $N(0, 5 \mu m)$ along the Z-axis feed motion $z$,

(4) Straightness deviations in Z-axis $\delta_x(z)$ and $\delta_y(z)$ in Eq. (3-14) and (3-24) are given randomly based on the normal distribution $N(0, 1 \mu m)$ along the Z-axis feed motion $z$, and

(5) Positioning and straightness deviations in X-axis in Eq. (3-14) and (3-24) are set to be 0, since the X-axis are fixed in both the boring and the turning processes.

![Fig. 4-10 2D roughness profile is evaluated by measuring the lines for bored faces](image1)

![Fig. 4-11 3D surface roughness is evaluated by measuring the areas for bored faces](image2)
Fig. 4-12 2D roughness profile is evaluated by measuring the lines for turned faces.

Fig. 4-13 3D surface roughness is evaluated by measuring the areas for turned faces.

Figure 4-10 and 4-11 shows the smallest value of $Sa$ and $Sq$ surface roughness from Table 4-2 of the 2D roughness profile which is evaluated by measuring the lines for bored faces, and 3D surface roughness is evaluated by measuring the areas for bored faces. The estimated values are used for the machining conditions of $f = 0.1$ mm/rev and $r_e = 1.6$ mm.
Figure 4-12 and 4-13 shows the smallest value of $Sa$ and $Sq$ surface roughness from Table 4-3 of the 2D roughness profile which is evaluated by measuring the lines for turned faces, and 3D surface roughness is evaluated by measuring the areas for turned faces. The estimated values are used for the machining conditions of $f = 0.1$ mm/rev and $r_e = 1.2$ mm.

4.6 Conclusion

A systematic method is proposed here to estimate both the 2D and 3D surface roughness through the boring and turning process simulations with kinematic motion deviations. The followings are concluded.

(1) A model is proposed to represent the kinematic motions of the cutting edges against the workpieces, taking into consideration of the kinematic deviations of the milling machines and the turning machines.

(2) The proposed model was applied to the simulation of simple boring and turning processes. The geometries of the bored and turned faces were obtained, based on the cutting conditions, the tool geometries and the kinematic deviations of the boring and turning processes.

(3) A systematic method is proposed to estimate the 2D and 3D surface roughness of the bored and turned faces based on the kinematic motions of the cutting edges, based on both the ISO and JIS standards dealing with the surface roughness evaluations. The proposed method provides us with a systematic method to verify and to evaluate the surface roughness based on the least mean square cylinders.

(4) The proposed method was applied to the estimation of both the 2D and 3D surface roughness, based on the simulation results of both the boring and turning processes. In the cases of the simulation without kinematic motion deviations, the results were same as the theoretical values. The disturbances in the surface roughness were numerically evaluated in the simulation results with kinematic motion deviations.
Chapter 5

Estimation of 3-Dimensional Tolerances Including Kinematic Motion Deviations
5. Estimation of 3-Dimensional Tolerances Including Kinematic Motion Deviations

5.1 Introduction

With advances of digital engineering technologies, CAD/CAE/CAM systems are now being widely applied to design, analysis and manufacturing processes of mechanical products. However, emphasis has been given mainly to the design, the analysis and the manufacturing of the nominal geometric shapes of the products.

One of the most important revolutions in the later part of the last century is introduction of manufacturing processes of mechanical product by using CNC machine tools, which are able to carry out various complicated machining processes without human interaction. Various types of CNC machine tools are now being designed and applied to machining processes of complicated machine products. The machining accuracy and the geometric dimensioning and tolerancing is one of the most important characteristics of the CNC machine tools for generating products with high accuracy and complicated geometries. Therefore, the dimensional tolerances and the geometric tolerances of the product shapes are very important from the viewpoints of the manufacturing process control and the quality control of the products. Much research has been carried out dealing with the analysis of the dimensional tolerances in the two- and/or three-dimensional shapes of the mechanical products, as shown in the review of relevant literature of chapter 2.

The dimensional tolerances represent only the sizes of the geometric features or the distances between a pair of the geometric features, such as the planes, the lines and the points in the shape models, and may not be sufficient for representation of the tolerances of the 3D complicated shapes. The geometric tolerances describe the allowable areas named “tolerance zones” which constrain the positions and the orientations of the geometric features in the 3D space, and are suitable to represent the tolerances of the 3D complicated shapes.

This chapter presents the estimation of 3D tolerances including kinematic motion deviations of machining centers based on geometric tolerances and estimated on the virtual part profiles as shown in Fig. 5-1. The key components of estimation of such deviations are:

- Geometric tolerances and deviations of features
- Kinematic motions deviations of linear and rotary tables based on geometric tolerances
- Analysis of kinematic motion deviations of rotary tables based on geometric tolerances
- Analysis of kinematic motion deviations of five-axis machining centers based on geometric tolerances
- Estimation of 3D tolerances based on simulation of virtual machining including kinematic motion deviations
Details of these features are provided below:

5.2 Geometric Tolerances and Deviations of Features (Sugimura, et al., 2003; Satonaka, et al., 2005b)

5.2.1 Features for geometric tolerances

The geometric tolerances represent the constraints on the positions and orientations of the geometric features, such as the planes, cylinders and axes of the products. In the followings, the geometric features are simply called features.

ISO/TC 213 defines several types of features as illustrated for a cylinder in Fig. 5-2 (ISO/TS 17450:1999). A truncated portion of a nominal geometric model of a product is called the “nominal integral feature”. Its axis is named the “nominal derived feature” to highlight the fact that the axis is derived from a cylindrical surface that is integral to the product boundary.

An actual realization of the products will have a surface that only approximately corresponds to the cylindrical feature, and is called the “real integral feature”. The real integral feature has an infinite number of points, and for the purpose of measurements only a finite subset of these will be used. The finite number of points sampled on the real integral surface forms what is called the “extracted integral feature”. Then, a perfect-form surface, such as a full cylinder, is fitted to the
sampled points to form what is termed the “associated integral feature”. In ISO/TC213, the term “association” is used synonymously with fitting. Note that the associated integral feature is a surface of the same type (e.g. cylinder) as the surface that contains the nominal integral feature. Finally, “associated derived feature” is the axis or other appropriate entity that is derived from the fitted surface.

This research aims to establish systematic methods for the analysis and the design of the geometric tolerances in the product design phase, in which the real products have not yet been manufactured. Therefore, only the associated integral and derived features, which have same geometric shapes as the nominal integral and derived features, are considered here.

5.2.2 Tolerance zones and deviation parameters

The geometric tolerances describe the allowable areas named “tolerance zones” which constrain the positions and orientations of the associated features in the 3D space. Figures 5-2, 5-3 and Table 5-1 summarize the various types of geometric tolerances and their tolerance zones, in particular the geometric tolerances for the 3D product model. In Table 5-1, the tolerance zones specify the alphabetical numbers of the shapes in Fig. 5-3.

The tolerances from the straightness to the profile of surface in Table 5-1 give the allowable areas of the single feature. The others represent the allowable areas of the features against the reference features named “datum”.

Fig. 5-2 Definition of features (ISO/TS 17450:1999)
Fig. 5-3 Types of tolerance zones

Table 5-1 Geometric tolerances and their tolerance zones

<table>
<thead>
<tr>
<th>Geometric Tolerance</th>
<th>Tolerance Zone</th>
</tr>
</thead>
<tbody>
<tr>
<td>Straightness</td>
<td>(b), (d), (e)</td>
</tr>
<tr>
<td>Flatness</td>
<td>(d)</td>
</tr>
<tr>
<td>Roundness</td>
<td>Between two circles</td>
</tr>
<tr>
<td>Cylindricity</td>
<td>(c)</td>
</tr>
<tr>
<td>Profile of line</td>
<td>Between two lines</td>
</tr>
<tr>
<td>Profile of surface</td>
<td>(d)</td>
</tr>
<tr>
<td>Parallelism</td>
<td>Line feature and datum line</td>
</tr>
<tr>
<td></td>
<td>(b), (d), (e)</td>
</tr>
<tr>
<td></td>
<td>Line feature/plan feature and datum plane</td>
</tr>
<tr>
<td></td>
<td>Plane feature and datum line</td>
</tr>
</tbody>
</table>
Perpendicularity
Line feature/plane feature and datum line
Line feature and datum plane (b), (d), (e)
Plane feature and datum plane (d)

Angularity
Line feature and datum line (d)
Line feature and datum plane (d)
Plane feature and datum line/datum plane (d)

Position
Point feature (a)
Line feature (b), (d), (e)
Plane feature (d)

Concentricity
Centre axis (b)

Symmetricity
Centre plane (d), (e)

<table>
<thead>
<tr>
<th>Table 5-2 Deviation parameters of tolerance features</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>(a) In a sphere</td>
</tr>
<tr>
<td>(b) In a cylinder</td>
</tr>
<tr>
<td>(c) Between two concentric cylinders</td>
</tr>
<tr>
<td>(e) Between two planes</td>
</tr>
<tr>
<td>(d) In rectangular parallelepipeds</td>
</tr>
</tbody>
</table>

$u$, $v$, $w$ : Position parameters, $u$, $w$ : Orientation parameters

The associated features obtained from the manufactured products have some position and orientation errors against the nominal features. The deviations of the associated features from the nominal features are represented by sets of parameters named “deviation parameters.” Table 5-2 summarizes the deviation parameters of the associated features for the individual types of the tolerance zones. For example, one location parameter $w$ and two orientation parameters $u$ and $v$ are required to represent the geometric deviations of the associated plane features against the nominal plane features for the case where the tolerance zone is given by the area between a pair of parallel planes.
5.2.3 Statistical properties of deviation parameters

5.2.3.1 Distribution of deviation parameters

Statistical methods are in general applied to the manufacturing process control and the quality control of the mechanical products. The measurements of the manufacturing operations and the products output from the operations are usually assumed to be the statistical variables following the normal distribution.

The deviations of the positions and orientations of the tolerance features of the products are generated through the manufacturing processes, therefore, the deviation parameters shown in Table 5-2 are assumed to be the statistical variables following the normal distributions.

The following assumptions are made in the analysis of the statistical properties of the deviation parameters.

(1) The deviation parameters $\delta_i$ representing the positions and orientations of the associated features follow the normal distribution $N(\mu_i, \sigma_i)$, and $\mu_i = 0$. Where, $\mu_i$ and $\sigma_i$ are the mean values and the standard deviations, respectively.

(2) The deviation parameters are independent of each other.

(3) The processes of manufacturing the products are well controlled, and the proportion of the non-conforming products represented by $P_d$, or “percent defective, is small. Here, the non-conforming products mean the products, in which the tolerance features exceed the tolerance zones.

5.2.3.2 Estimation of standard deviations

The issue discussed here is to estimate the standard deviations of the deviation parameters of the tolerance features, which satisfy the specified percent defective $P_d$.

Consider a case where a set of deviation parameters of a tolerance feature and their standard deviations are represented by $\delta_i$ and $\sigma_i$ $(i=1,2,...,n)$, respectively. The following equation is assumed to represent the relationship between the standard deviations $\sigma_i$ and the width of the tolerance zone.

$$\sigma_i = \frac{\delta_{\text{max}}}{C_{P_d}} \quad (5-1)$$

where,

$\delta_{\text{max}}$ : maximum value of the deviation parameters $\delta_i$, if the other deviation parameters $\delta_j = 0, (i \neq j)$. Since the tolerance zone is symmetry and the mean value $\mu_i = 0$, the minimum value of the deviation parameter is $-\delta_{\text{max}}$. 
$C_{Pd}$: a constant representing the ratio of the maximum values $\delta_{\text{max}}$ and the standard deviation $\sigma_i$.

The probability that the tolerance features are included in the tolerance zone is given by the following equation based on the deviation parameters $\delta_i$.

$$1 - Pd = 2^n \int_0^{\delta_{\text{max}}} \int_0^{f(\delta_{\text{max}}, \delta_i)} \int_0^{f(\delta_{\text{max}}, \delta_i, \delta_j)} \cdots \left( \prod_{i=1}^{n} G_{\delta_i} (\delta_i) \right) d\delta_n \cdots d\delta_2 d\delta_1$$

(5-2)

where, the functions $f$ represent the allowable ranges of the parameters $\delta_i$, which are constrained by the other parameters $\delta_j$.

If the parameters $\delta_i$ are replaced by the following normalized parameters $x_i$, Eq. (5-2) is transformed into Eq. (5-4).

$$\delta_i = \sigma_i x_i = \frac{\delta_{\text{max}}}{C_{Pd}} x_i, \quad d\delta_i = \frac{\delta_{\text{max}}}{C_{Pd}} dx_i$$

(5-3)

$$1 - Pd = \left( \frac{2}{\sqrt{2\pi}} \right)^n \int_0^{x_{\text{max}}} \int_0^{g(x, x_{\text{max}}, x_i)} \int_0^{g(x, x_{\text{max}}, x_i, x_j)} \cdots \left( \prod_{i=1}^{n} \exp \left( -\frac{x_i^2}{2} \right) \right) dx_n \cdots dx_2 dx_1$$

(5-4)

where, $g$ is the transformed function of $f$ in Eq. (5-2).

Let us consider a case shown in Fig. 5-4, as an example. The maximum values $\delta_{\text{max}}$ are given as follows.

![Fig. 5-4 Definition of geometric tolerance of plane features](image-url)
\[ \delta_1 = w, \delta_2 = \alpha, \delta_3 = \beta \]
\[ \delta_{1\text{max}} = t/2, \delta_{2\text{max}} = t/L_1, \delta_{3\text{max}} = t/L_2 \] (5-5)

where,
\( L_1, L_2 \) : Length and width of the plane feature.
\( t \) : Tolerance values, e.g. the distance between two planes representing the tolerance zones.

The following equation gives the conditions that the plane features are included within the tolerance zone between a pair of planes.

\[ -\frac{t}{2} \leq \delta_1 + \frac{L_1}{2} \delta_2 + \frac{L_2}{2} \delta_3 \leq \frac{t}{2} \] (5-6)

The probability that the tolerance features are included within the tolerance zones is given by the following equation by transformed function of Eq. (5-2) and (5-4).

\[ 1 - Pd = \left( \frac{2}{\sqrt{2\pi}} \right)^3 \int_0^{C_{pd}} \int_0^{C_{pd}} \int_0^{C_{pd} - x_1 - x_2} \left( \prod_{i=1}^{3} \exp \left( -\frac{x_i^2}{2} \right) \right) dx_1 dx_2 dx_3 \] (5-7)

where,
\[ x_1 = \frac{2C_{pd} \delta_1}{t}; \quad x_2 = \frac{L_1C_{pd} \delta_2}{t}; \quad x_3 = \frac{L_2C_{pd} \delta_3}{t} \]

If the percent defective \( Pd \) is set to be 0.27%, the constant \( C_{pd} \) can be estimated as “\( C_{pd} = 5.83 \)” through the numerical analysis of Eq. (5-7). The standard deviations \( \sigma_i \) of deviation parameters \( \delta_i \) are also obtained based on the \( C_{pd} \), the dimensions of the plane features and the size of tolerance zone \( t \), by applying Eqs. (5-1) and (5-5).

![Fig. 5-5 Definition of geometric tolerance of axis features](image)
In the case of an axis feature shown in Fig. 5-5, the following equation gives the deviation parameter and their maximum values. Figure 5-5, where a centerline of a cylinder is constrained by a cylinder. The maximum values of the deviation parameters and the standard deviations of the parameters are given in the following equations.

\[ \delta_1 = u, \ \delta_2 = v, \ \delta_3 = \alpha, \ \delta_4 = \beta \]

\[ \delta_{1\text{max}} = \delta_{2\text{max}} = \frac{t}{2}; \ \delta_{3\text{max}} = \delta_{4\text{max}} = \frac{t}{\ell}; \]  

(5-8)

The constraints on the deviation parameters of the centerline are given as follows, in order that the centerline is included within the tolerance zone

\[ \left( \frac{\ell}{2} \beta + u \right)^2 + \left( \frac{\ell}{2} \alpha + v \right)^2 \leq \left( \frac{t}{2} \right)^2, \ \left( \frac{\ell}{2} \beta - u \right)^2 + \left( \frac{\ell}{2} \alpha - v \right)^2 \leq \left( \frac{t}{2} \right)^2 \]  

(5-9)

In this case, the \( C_{pd} \) is estimated as \( "C_{pd} = 5.06," \) if the percent defective \( Pd \) is set to be \( 0.27\% \). The \( C_{pd} \) value obtained here is same in the case that a cylindrical feature is constrained by a pair of co-axial cylinders. The \( C_{pd} \) values for plane features and axis features obtained here will be applied for the following sections.

5.3 Kinematic Motions Deviations of Linear Tables and Rotary Tables Based on Geometric Tolerances

5.3.1 Geometric deviations of components and kinematic motion deviations of machine tools

The shape generation processes of the machine tools are carried out by the shape generation motions between the tools and the workpieces, and the shape generation motions are realized by a set of the relative motions between the constituting components. Therefore, the kinematic motion deviations of the shape generation motions are influenced by the geometric deviations of the components and the kinematic motion deviations between these components.

Figure 5-6 summarizes the relationships between the kinematic motion deviations of the shape generation motions and the deviations of the components. In the figure, the nodes WO, CT and \( F_{ij} \) represent the workpieces to be machined, the tools, and the geometric features connecting the rigid components Unit-i of the machine tools, respectively. The features mean here the geometric elements, such as plane faces of guide ways and cylindrical faces of bearings, which connect and constrain the relative motions between the pairs of the units.

The kinematic motion deviations of the shape generation motions of the machine tools are defined as the kinematic motion deviations between the tools CT against the workpieces WO, and
the deviations are influenced by various types of deviations, such as the position deviations in setting up the tools and the workpieces, the relative position deviations between the units, and the geometric deviations of the features connecting the constituent units.

This present research is to represent and to analyze the kinematic motion deviations of the tools against the workpieces, based on the geometric deviations of the components. In particular, emphasis is given to modeling and analysis of the effects of the geometric deviation of the features on the kinematic motion deviations of the tools against the workpieces.

### 5.3.2 Geometric deviation of linear and rotary tables

Figure 5-7 (a) shows a typical example of the linear tables utilized for the machine tools. A base supports and guides a table by four plane faces called guide-ways, which are indicated by \( a, b, c \) and \( d \). Figure 5-7 (b) shows a typical example of rotary tables with a vertical axis utilized for the machine tools. A base supports and guides a rotary table by a plane face and a cylindrical face called guide-ways for rotary tables, which are indicated by \( a \) and \( b \). In the case, the relative position and orientation of the table against the base are given by the following equation, if the guide-ways coincide with the nominal features.
(a) Linear table with vertical axis (Satonaka, et al., 2007b)

(b) Vertical rotary tables (C-axis table) and their guide-ways

(c) Guide-ways between table and base

Fig. 5-7 Linear and rotary tables and their connecting relationships

\[ x_i = A_d A_a^{-1} x_{i+1} = A_b A_b^{-1} x_{i+1} \]  \hspace{1cm} (5-10)

where,

\[ x_i \] : Position vector of a point \( P \) in the base coordinate system \( C_i \)
\[ x_{i+1} \] : Position vector of \( P \) in the table coordinate system \( C_{i+1} \).
A_j (j = a, ..., d): 4 x 4 homogeneous transformation matrices representing the positions and orientations of the guide ways j in the base coordinate system C_i.

A_{j+1} (j = a, ..., d): 4 x 4 homogeneous transformation matrices representing the positions and orientations of the guide ways j in the table coordinate system C_{i+1}.

However, the relative positions of the table against the base are influenced by the position and orientation deviations of the guide ways, if the guide ways do not coincide with the nominal features. The positions of the table against the base are represented by the following equation corresponding to the individual guide ways.

\[ x_i = A_j D_j D_{j'}^{-1} A_{j'}^{-1} x_{i+1} \quad (j = a, ..., d) \]  

(5-11)

where,

\[ D_j, D_{j'} : 4 \times 4 \] homogeneous transformation matrices representing the position and orientation deviations of the pair of features constituting the guide ways j.

Equation (5-11) means that various values of the relative position of the table against the base are obtained for the individual guide ways, as shown in Fig. 5-7 (c). A systematic method is therefore required to evaluate one value of the relative position for the cases where the guide ways have the geometric deviations. It is because that both the base and the table are the rigid bodies and the relative position between the rigid bodies should be represented by one value. The following two methods are proposed here to solve the problem.

(1) Priority among connections

Let us consider the case shown in Fig. 5-8 (a) as an example. The Unit-i and the Unit-i+1 is connected by two set of the features indicated by A and B, which are perpendicular to each other. The deviations of the relative positions of the Unit-i+1 to the Unit-i, which are specified by \( D_j D_{j'}^{-1} \) in Eq. (5-11), are given by the following equations for the individual sets of the connecting features.

(a) Priority between connection features
(b) No priority between connecting feature

Fig. 5-8 Features connecting a pair of unit
In this case, two values are obtained for defining the relative positions of the Unit-i and the Unit-i+1. Priority may be set to the connecting features, based on the areas of the features and the direction of the gravity. Higher priority given to the feature set $A$ in the case, if the direction of the gravity is $-Z$. Therefore, the deviations of the relative positions of the Unit-i+1 to the Unit-i are given as follows.

\[
D_A D_A^{-1} = \begin{bmatrix}
1 & 0 & \delta_{\beta A} & 0 \\
0 & 1 & -\delta_{\alpha A} & 0 \\
-\delta_{\beta A} & \delta_{\alpha A} & 1 & \delta_{w A} \\
0 & 0 & 0 & 1
\end{bmatrix}
\] (5-12)

\[
D_B D_B^{-1} = \begin{bmatrix}
1 & -\delta_{\beta B} & \delta_{\beta B} & \delta_{w B} \\
\delta_{\beta B} & 1 & 0 & 0 \\
-\delta_{\beta B} & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(2) Non-priority among connections

For the case shown in Fig. 5-8 (b), there are two sets of the connecting features between the Unit-i and the Unit-i+1. However, there is not any priority between the feature sets $A$ and $B$. In this case, the deviations for the both connecting feature sets are averaged based on the least square method (Satonaka, et al., 2008), and the deviations of the relative position of the Unit-i+1 to the Unit-i are given as follows.

\[
D_{AB} D_{AB}^{-1} = \begin{bmatrix}
1 & -\delta_{\beta B} & \delta_{\beta A} & \delta_{w B} \\
\delta_{\beta B} & 1 & -\delta_{\alpha A} & 0 \\
-\delta_{\beta A} & \delta_{\alpha A} & 1 & \delta_{w A} \\
0 & 0 & 0 & 1
\end{bmatrix}
\] (5-13)

where, “$A > B$” specifies that the feature set $A$ has higher priority to the feature set $B$. 

(2) Non-priority among connections

For the case shown in Fig. 5-8 (b), there are two sets of the connecting features between the Unit-i and the Unit-i+1. However, there is not any priority between the feature sets $A$ and $B$. In this case, the deviations for the both connecting feature sets are averaged based on the least square method (Satonaka, et al., 2008), and the deviations of the relative position of the Unit-i+1 to the Unit-i are given as follows.
where, “A = B” means that the feature sets A and B do not have any priority.

5.3.3 Modeling of kinematic motion deviations of linear tables (Satonaka, et al., 2007b)

The geometric deviations of the linear tables are estimated based on the geometric deviations of the guide-ways of a, b, c, and d in Fig. 5-7 (a). Equation (5-15) is obtained to describe the motion deviations of the linear tables, under the assumption that the priority among the guide-ways is “(a = c) > (b = d).” In Eq. (5-15), y is the relative linear motions of the tables against the bases.

\[
\begin{align*}
\mathbf{D}_{AB}^{-1} \mathbf{D}_{AB}' &= \\
&= \begin{bmatrix}
1 & 0 & \frac{1}{2}(\delta_{\beta A} + \delta_{\beta B}) & 0 \\
0 & 1 & \frac{1}{2}(\delta_{\alpha A} + \delta_{\alpha B}) & 0 \\
\frac{1}{2}(\delta_{\beta A} + \delta_{\beta B}) & \frac{1}{2}(\delta_{\alpha A} + \delta_{\alpha B}) & 1 & \frac{1}{2}(\delta_{wA} + \delta_{wB}) \\
0 & 0 & 0 & 1
\end{bmatrix}
\end{align*}
\]

(5-14)

where, \( \mathbf{A}^2(y) = \)

\[
\begin{align*}
\begin{bmatrix}
1 & -\frac{\delta_{\gamma b} + \delta_{\gamma d}}{2} & \frac{\delta_{\beta A} + \delta_{\beta c}}{2} & \frac{1}{2}\left[(\delta_{\gamma a} + \delta_{\gamma d}) - (\delta_{\beta d} + \delta_{\gamma d})y\right] \\
\frac{\delta_{\gamma b} + \delta_{\gamma d}}{2} & 1 & -\frac{\delta_{\alpha A} + \delta_{\alpha c}}{2} & y \\
\frac{\delta_{\beta A} + \delta_{\beta c}}{2} & \frac{\delta_{\alpha A} + \delta_{\alpha c}}{2} & 1 & \frac{1}{2}\left[(\delta_{\gamma a} + \delta_{\gamma c}) + \frac{1}{2}(\delta_{\alpha b} + \delta_{\beta b}) + (\delta_{\alpha d} + \delta_{\gamma d})y\right] \\
0 & 0 & 0 & 1
\end{bmatrix}
\end{align*}
\]

(5-15)

where,

- \( y \) : y-axis direction travel.
- \( \delta_{pq} \) : Differences of the position and orientation deviations of the features in base side and table side consisting of the \( q \)th guide-ways of in the \( p \)th directions.
- \( p (= \alpha, \beta, \gamma, u, v, w) \) : Components of the position and orientation deviations.
- \( q (= a, b, c, d) \) : Guide-ways.
5.3.4 Modeling of kinematic motion deviations of rotary tables

The geometric deviations of the vertical rotary tables shown in Fig. 5-7 (b) are formulated based on the geometric deviations of the guide ways, by applying the method described in section 5.3.2. Equation (5-16) is obtained to describe the kinematic motion deviations of the rotary table, on the assumption that the priority among the guide-ways is “a > b.” Where, “a > b” means that the higher priority is given to the guide-way a against b to determine the positions and orientations of the tables to the bases considering the geometric deviations of the guide-ways.

\[
\mathbf{A}^6(\theta) = \begin{bmatrix}
\cos \theta & -\sin \theta & \delta_{y_{b1}} & \delta_{x_{b}} \\
\sin \theta & \cos \theta & \delta_{v_{b1}} & \delta_{y_{b}} \\
\delta_{y_{b2}} & \delta_{v_{b2}} & 1 & \delta_{z_{b}} \\
0 & 0 & 0 & 1 
\end{bmatrix}
\]  

(5-16)

where,
\[
\theta \quad : \text{Rotational angle of the table.} \\
\delta_{va1} = -\beta_{b2} \sin \theta + \alpha_{b2} \cos \theta - \alpha_{b1} \\
\delta_{va2} = \beta_{b1} \sin \theta + \alpha_{b1} \cos \theta - \alpha_{b2} \\
\delta_{y_{b1}} = -\beta_{b2} \cos \theta - \alpha_{b2} \sin \theta + \beta_{b1} \\
\delta_{y_{b2}} = -\beta_{b1} \cos \theta + \alpha_{b1} \sin \theta + \beta_{b2} \\
\delta_{x_{b}} = -l_{1 \beta_{b2}} \cos \theta - l_{1 \alpha_{b2}} \sin \theta + l_{1 \beta_{b1}} - \delta_{c_{b2}} \cos \theta + \delta_{y_{b2}} \sin \theta + \delta_{x_{b1}} \\
\delta_{y_{b}} = -l_{1 \beta_{b2}} \sin \theta + l_{1 \alpha_{b2}} \cos \theta - l_{1 \beta_{b1}} - \delta_{c_{b2}} \sin \theta - \delta_{y_{b2}} \cos \theta + \delta_{y_{b1}} \\
\delta_{z_{b}} = \delta_{c_{a1}} - \delta_{a2} \\
\alpha_{ip}, \beta_{ip}, \gamma_{ip} \quad : \text{Orientation deviations the jth geometric feature of guide-way i around x, y and z axis of the rotary tables.} \\
\delta_{x_{ij}}, \delta_{y_{ij}}, \delta_{z_{ij}} \quad : \text{Position deviations the jth geometric feature of guide-way i along X, Y and Z axis of the rotary tables.}
\]

For the cases of the horizontal rotary tables shown in Fig. 5-9, the kinematic motion deviations are formulated in Eq. (5-17), under the assumption that the priority among the guide-ways is “(c = e) > (d = f).” Where, “(c = e)” means that there is not any priority between the guide-ways c and e to determine the positions and orientations of the tables to the bases considering the geometric deviations of the guide-ways.
Fig. 5-9 Horizontal rotary tables (A-axis table) and their guide-ways

\[ A^4(\varphi) = \begin{bmatrix}
1 & \frac{1}{2} \delta_{Hy1} & \frac{1}{2} \delta_{Hy1} & \frac{1}{2} \delta_{Hz} \\
\frac{1}{2} \delta_{Hy2} & \cos \varphi & -\sin \varphi & \frac{1}{2} \delta_{Hy} \\
\frac{1}{2} \delta_{Hy2} & \sin \varphi & \cos \varphi & \frac{1}{2} \delta_{Hz} \\
0 & 0 & 0 & 1
\end{bmatrix} \]

(5-17)

where,

\( \varphi \) : Rotational angle of the table.

\[
\begin{align*}
\delta_{Hy1} &= -\beta_{c2} - \beta_{c2} + (\gamma_{e1} + \gamma_{e1}) \sin \varphi + (\beta_{c1} + \beta_{c1}) \cos \varphi \\
\delta_{Hy2} &= -\beta_{c1} - \beta_{c1} - (\gamma_{e2} + \gamma_{e1}) \sin \varphi + (\beta_{c2} + \beta_{c2}) \cos \varphi \\
\delta_{Hy} &= \gamma_{e2} + \gamma_{e2} - (\gamma_{e1} + \gamma_{e1}) \cos \varphi + (\beta_{c2} + \beta_{c2}) \sin \varphi \\
\delta_{Hz} &= \delta_{x1} - \delta_{x2} + \delta_{y1} - \delta_{y2} \\
\delta_{Hy} &= \delta_{x1} + \delta_{x1} - (\delta_{y2} + \delta_{y2}) \cos \varphi + (\delta_{x2} + \delta_{x2}) \sin \varphi \\
\delta_{Hz} &= \frac{1}{2} \left\{ (\gamma_{e1} - \gamma_{e1}) - (\gamma_{e2} - \gamma_{e2}) \cos \varphi - (\beta_{c2} - \beta_{c2}) \sin \varphi \right\} \\
\delta_{Hz} &= \frac{1}{2} \left\{ (\beta_{c1} - \beta_{c1}) - (\beta_{c2} - \beta_{c2}) \cos \varphi + (\gamma_{e2} - \gamma_{e2}) \sin \varphi \right\}
\end{align*}
\]
5.4 Analysis of Kinematic Motion Deviations of Rotary Tables

A two-axis rotary table shown in Fig. 5-10 is designed by combining two tables shown in Figs. 5-7 (b) and 5-9, and the kinematic motion deviations of the table are estimated based on the formulas presented in Eqs. (5-16) and (5-17). The important conditions of the tables are assumed as follows:

1. Rotary table with vertical axis
   - Table sizes in Fig. 5-7 (b):
     \( l_1 = 70 \) [mm], \( R_1 = 250 \) [mm]
   - Standard deviations of all the deviation parameters \( \delta \) in Eq. (5-16).
     Position deviations = \( 1 \times 10^{-3} \) [mm]
     Orientation deviations = \( 1 \times 10^{-3} \) [rad]

2. Rotary table with horizontal axis
   - Table sizes in Fig. 5-9:
     \( l_2 = 1,000 \) [mm], \( l_3 = 500 \) [mm]
   - Standard deviations of all the deviation parameters \( \delta \) in Eq. (5-17).
     Position deviations = \( 1 \times 10^{-3} \) [mm]
     Orientation deviations = \( 1 \times 10^{-3} \) [rad]

The kinematic motion deviation parameters in Eqs. (5-16) and (5-17) are given by both the mean values and the standard deviations, therefore, all the deviation parameters in the transformation matrices are estimated by applying the following equations.

\[
\mathbf{C} = \mathbf{AB}, \quad \mathbf{C} = \{c_{ij}\}, \mathbf{A} = \{a_{ij}\}, \mathbf{B} = \{b_{ij}\}
\]  \hspace{1cm} (5-18)
\[
\mu_{ij} = \sum_{k=1}^{4} \mu_{ik}^a \mu_{kj}^b
\]

\[
\sigma_{ij} = \sqrt{\sum_{k=1}^{4} \left( (\mu_{ik}^a \sigma_{kj})^2 + (\mu_{kj}^b \sigma_{ik})^2 \right) }
\]

where,
\[
\mu_{ij}^a, \mu_{ik}^a, \mu_{kj}^b : \text{Mean values of the elements of transformation matrices A, B and C.}
\]
\[
\sigma_{ij}^c, \sigma_{ik}^a, \sigma_{kj}^b : \text{Standard deviations of the elements of transformation matrices A, B and C.}
\]

Fig. 5-11 Mean values and standard deviations of kinematic motion of point P (Standard deviations are magnified by 100)

Kinematic motion deviations of a point P in Fig. 5-10 are estimated by applying Eqs. (5-16), (5-17) and (5-18). The point P is selected as a point that clearly presents the effects of the kinematic motion deviations in the rotational motion around the X and Z axes. The initial position of P is set that P coincides with the rotational axis of X axis and that the rotational radius around the Z axis is largest. The kinematic deviations in X-Y plane and ones in X-Z plane are presented in Figs. 5-11 (a) and (b), respectively. In the figure, the crossing points of a pair of line segments give the mean values of the kinematic motion of the point P, and the vertical and horizontal line segments show the standard deviations in X-, Y- and Z-directions of the kinematic motion of the point P. The standard deviations are magnified by 100 for the ease of understand. It is understood that the kinematic motion deviations are evaluated based on the mean values and the standard deviations in X-, Y- and Z-directions.
The kinematic motion deviations of any points on the table are evaluated by applying the proposed methods. In the next section, the model is expanded to represent the five-axis machine tools, aimed at analyzing the kinematic motion deviations of the cutting points on the tools against the workpieces.

5.5 Analysis of Kinematic Motion Deviations of Five-Axis Machining Centers

5.5.1 Modeling of kinematic motions

The five-axis machining centers are composed of two rotary tables and three linear tables shown in Fig 5.12. Three types of five–axis machining centers shown in Fig 5.12 are considered here for the analysis. The shape generation motions of the machining centers are summarized in Eq. (5-19) as shown below.

(a) Type 1: Workpiece rotation type
(b) Type 2: Tool & workpiece rotation type
(c) Type 3: Tool rotation type

Fig. 5-12 Five-axis machining centers

Type 1: \[ \mathbf{r}_w = \mathbf{A}^{3}(d_1)\mathbf{A}^{6}(\theta)\mathbf{A}^{3}(d_2)\mathbf{A}^{4}(\varphi)\mathbf{A}^{3}(d_3)\mathbf{A}^{1}(x)\mathbf{A}^{3}(d_4)\mathbf{A}^{2}(y)\mathbf{A}^{3}(d_5)\mathbf{A}^{3}(z)\mathbf{r}_r \]

Type 2: \[ \mathbf{r}_w = \mathbf{A}^{3}(d_1)\mathbf{A}^{6}(\theta)\mathbf{A}^{3}(d_2)\mathbf{A}^{1}(x)\mathbf{A}^{3}(d_4)\mathbf{A}^{2}(y)\mathbf{A}^{3}(d_5)\mathbf{A}^{3}(z)\mathbf{A}^{3}(d_5)\mathbf{A}^{3}(\varphi)\mathbf{r}_r \]

Type 3: \[ \mathbf{r}_w = \mathbf{A}^{3}(d_1)\mathbf{A}^{1}(x)\mathbf{A}^{3}(d_2)\mathbf{A}^{2}(y)\mathbf{A}^{3}(d_3)\mathbf{A}^{3}(z)\mathbf{A}^{3}(d_4)\mathbf{A}^{6}(\theta)\mathbf{A}^{3}(d_5)\mathbf{A}^{4}(\varphi)\mathbf{r}_r \]

where,
\[ \mathbf{A}^{3}(x) : \text{X-axis linear motion} \]
\[ \mathbf{A}^{2}(y) : \text{Y-axis linear motion} \]
\( A^3(z) \): Z-axis liner motion
\( A^4(\varphi) \): A-axis rotary motion
\( A^5(\theta) \): C-axis rotary motion
\( A^3(d_i) \): Translation between the tables
d_i : Distances in Z-direction between pairs of constituting tables. They are summarized in Table 5-3.

| Table 5-3 Distances between a pair of connecting tables |
|-----------------|-----------------|-----------------|
| Sizes [mm]      | Type 1          | Type 2          | Type 3          |
| \( d_1 \)       | -20             | -20             | -30             |
| \( d_2 \)       | 60              | -90             | -70             |
| \( d_3 \)       | -230            | -70             | 390             |
| \( d_4 \)       | -70             | 400             | -70             |
| \( d_5 \)       | 450             | -30             | -30             |

### 5.5.2 Analysis of kinematic motion deviations of five-axis machining centers

The dimensions and the strokes shown in Table 5-4 according to Fig. 5-13 are given to X-, Y- and Z-tables for three types of machining centers. The dimensions are same for all types of machining centers. Table 5-5 summarizes the dimensions of the rotary tables, which are designed for the individual types of machining centers.

![Fig. 5-13 Sizes and strokes of linear and rotary tables](image)
Table 5-4 Distances between a pair of connecting tables

<table>
<thead>
<tr>
<th>Sizes [mm]</th>
<th>X-table</th>
<th>Y-table</th>
<th>Z-table</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_{a1}$</td>
<td>95</td>
<td>110</td>
<td>40</td>
</tr>
<tr>
<td>$l_{a2}$</td>
<td>12</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>$l_{b1}$</td>
<td>125</td>
<td>160</td>
<td>55</td>
</tr>
<tr>
<td>$l_{c1}$</td>
<td>95</td>
<td>110</td>
<td>40</td>
</tr>
<tr>
<td>$l_{a2}$</td>
<td>12</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>$l_{d1}$</td>
<td>65</td>
<td>60</td>
<td>25</td>
</tr>
<tr>
<td>$d_a$</td>
<td>60</td>
<td>100</td>
<td>30</td>
</tr>
<tr>
<td>$d_b$</td>
<td>30</td>
<td>40</td>
<td>60</td>
</tr>
<tr>
<td>$d_c$</td>
<td>60</td>
<td>100</td>
<td>30</td>
</tr>
<tr>
<td>$d_d$</td>
<td>30</td>
<td>40</td>
<td>60</td>
</tr>
<tr>
<td>Length [mm]</td>
<td>500</td>
<td>350</td>
<td>200</td>
</tr>
<tr>
<td>Stroke [mm]</td>
<td>$-200 \leq X \leq 200$</td>
<td>$-175 \leq Y \leq 175$</td>
<td>$0 \leq Z \leq 190$</td>
</tr>
</tbody>
</table>

Table 5-5 Sizes of rotary tables for types 1, 2 and 3

<table>
<thead>
<tr>
<th>Tables</th>
<th>Sizes [mm]</th>
<th>Types</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Types 1</td>
</tr>
<tr>
<td>C-axis table</td>
<td>$d_b$</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>$l_{j}$</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>$R_{j}$</td>
<td>102.5</td>
</tr>
<tr>
<td>A-axis table</td>
<td>$d_c (= d_e)$</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>$d_d (= d_f)$</td>
<td>$\phi$30</td>
</tr>
<tr>
<td></td>
<td>$l_{c1}$</td>
<td>340</td>
</tr>
</tbody>
</table>

Table 5-6 Table positions to be set in analysis

<table>
<thead>
<tr>
<th>Positions</th>
<th>X-axis $x$ [mm]</th>
<th>Y-axis $y$ [mm]</th>
<th>Z-axis $z$ [mm]</th>
<th>A-axis $\phi$ (degree)</th>
<th>C-axis $\theta$ (degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position-1</td>
<td>200</td>
<td>175</td>
<td>190</td>
<td>90</td>
<td>180</td>
</tr>
<tr>
<td>Position-2</td>
<td>100</td>
<td>90</td>
<td>95</td>
<td>45</td>
<td>90</td>
</tr>
<tr>
<td>Position-2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
The kinematic motion deviations of the tools against the workpieces are analyzed by applying Eq. (5-19), according to the following procedure.

STEP 1: Set the table positions

The table positions are set as shown in Table 5-6 to analyze the kinematic motion deviations for the individual positions.

STEP 2: Set the initial tolerance values for all the guide-ways

All the tolerance values are set that \( t = 50 \, \mu \text{m} \) for all the guide-ways shown in Fig. 5-13. These values are assumed as the initial values. The standard deviations of the position and orientation deviations of the tools against the workpieces are estimated by applying Eq. (5-19), based on the formulas given by Eq. (5-15), (5-16) and (5-17) representing the kinematic motions and their standard deviations of the linear and the rotary tables. The standard deviations of these table motions are also estimated, based on the standard deviations of the position and orientation deviations of the individual guide-ways given in Eq. (5-1).

STEP 3: Set the small tolerance value for each guide-way

A tolerance values of one of the guide-ways are set that \( t = 5 \, \mu \text{m} \), which is \( 1/10 \) of the initial values, in order to clarify the effects of the improvement of the tolerance values of individual guide-ways. The standard deviations of the position and orientation deviations of the tools against the workpieces are estimated in same manner shown in STEP 2, and the obtained standard deviations are compared by the following equation to clarify the effects of the individual guide-ways.

\[
ir = \frac{St_{50} - St_{5}}{St_{50}} \times 100
\]

where,

\( St_{50} \) : Standard deviations obtained based on initial tolerance values.

\( St_{5} \) : Standard deviations obtained based on improved tolerance values.

The \( ir \) values obtained here show the improvement ratio of the standard deviation of the kinematic motions between the tools against the workpieces, when one tolerance value is changed from \( t = 50 \, \mu \text{m} \) to \( t = 5 \, \mu \text{m} \). When the \( ir \) value of one guide-way is high, the improvement of the tolerance values of the guide-way is very effective to improve the whole kinematic deviations of the five-axis machining centers.

Figures 5-14, 5-15 and 5-16 summarize the analysis results of the \( ir \) values for all the table positions and all the types of the machining centers. It is found that the improvement ratio of the A-axis and C-axis rotary tables is higher than ones of the linear tables for all cases of the analysis conditions. In particular, the guide-ways \( e \) and \( e \) of A-axis and the guide-way \( a \) of C-axis are
larger effects than the other guide-ways, since these guide-ways have higher priority to determine the kinematic motion deviations for the rotary tables, as shown in Eqs. (5-16) and (5-17).

As regards the types of machining centers, the type 3 has different characteristics from the others. The improvement ratio of the x-axis and y-axis linear tables is relatively high against the other types of machining centers. The type 3 machining centers have no rotational axes in the workpiece side, and the effects of the rotational motion deviations in tool side may be reduced, since they have smaller arm lengths between the rotational center of the tool side and the cutting points of the tools.

It is understood that the proposed model is effective to analyze the effects of the tolerance values of the guide-ways to the kinematic motion deviations of the tools against the workpieces in the five-axis machining centers.
Fig. 5-14 Analysis results of type 1 machining center

(a) Position-1

(b) Position-2

(c) Position-3
Fig. 5-15 Analysis results of type 2 machining center

(a) Position-1

(b) Position-2

(c) Position-3
5.6 Estimation of 3D Tolerances Based on Simulation of Virtual Machining Including Kinematic Motion Deviations

5.6.1 Verification of 3-dimensional tolerances

Modeling of form errors associated with machined parts is very important for the high-precision manufacturing. The least-squares method is traditionally used to minimize the sum of the square errors for the part profile evaluation, because the form errors estimated by the least-squares method are not minimum (Lai, et al., 2000). Therefore, the current ASME Y14.5 defines a minimum zone between two offset envelope surfaces obtained from the actual measurement such that maximum deviation between the datum and the actual feature, concerned for evaluating the form errors (Endrias and Feng, 2003).

Assessment cylinders shown in Fig. 5-17 are considered here as the reference datum to measure the 3D tolerances of the bored and turned surfaces. The assessment cylinders are generated based on all the 3D coordinate data generated by the 3D boring and turning process simulations are described from chapter 3.

A procedure is proposed to generate an assessment cylinder in order to verify the 3D tolerances based on all the trajectories of the cutting edge in the boring and turning processes. The parameters to be determined here are summarized in the following, which represent a cylinder in the reference coordinate system $X_RY_RZ_R$. 

(c) Position-3

Fig. 5-16 Analysis results of type 3 machining center
Fig. 5-17 Coordinate system for measuring deviations

(1) Unit vector \((l_0, m_0, n_0)\) representing the direction of the axis of the assessment cylinder,  
(2) Vector \((a_0, b_0, 0)\) representing the position of the axis of the assessment cylinder, and  
(3) \(R_0\) representing the radius of the assessment cylinder.

The reference coordinate system \(X_RY_RZ_R\) coincides the ones in Figs. 3-7 and 3-8, therefore, the parameters \(l_0\) and \(m_0\) are fixed, the parameters \(a_0, b_0,\) and \(R_0\) are estimated by applying the following equations, based on the least mean square method (Thomas and Chan, 1989).

\[
a_0 = \frac{c_1b_2 - c_2b_1}{a_1b_2 - a_2b_1} \tag{5-21}
\]

\[
b_0 = \frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1} \tag{5-22}
\]

\[
R_0 = \frac{1}{N} \left\{ \sum_{i=1}^{N} X_i^2 - 2 \sum_{i=1}^{N} X_i a_0 + N a_0^2 + \sum_{i=1}^{N} Y_i^2 - 2 \sum_{i=1}^{N} Y_i b_0 + N b_0^2 \right\} \tag{5-23}
\]

where,

\[
a_i = 2 \left( \sum_{i=1}^{N} X_i \right)^2 - N \sum_{i=1}^{N} X_i^2 \quad b_i = 2 \left( \sum_{i=1}^{N} X_i \sum_{i=1}^{N} Y_i - N \sum_{i=1}^{N} X_i Y_i \right)
\]

\[
a_2 = 2 \left( \sum_{i=1}^{N} X_i \sum_{i=1}^{N} Y_i - N \sum_{i=1}^{N} X_i Y_i \right) \quad b_2 = 2 \left( \sum_{i=1}^{N} Y_i^2 \right)^2 - N \sum_{i=1}^{N} Y_i^2
\]
The distances from the measured \( R_i \) of all the generated point \( P_i( X_i, Y_i, Z_i ) \) \( (i = 1, 2, \ldots, N) \) and the assessment cylinder is given in the following equation, as shown in Fig. 5-17.

\[
R_i = \left( X_i - l_0 Z_i - a_0 \right)^2 + \left( Y_i - m_0 Z_i - b_0 \right)^2
\]  

(5-24)

According to the minimum zone solution defined in ISO 1101:2012 for a given set of data points \( P_i \), if all the data points \( P_i \) are on or between the two coaxial ideal cylinders, the minimal radial separation \( t \) between the two coaxial ideal cylinders is called the minimum zone cylindricity error shown in Fig. 5-17. It can be expressed as:

\[
t = f( a_0, b_0, l_0, m_0 ) \\
= min( max( R_i ) - min( R_i ) ) \\
= min( R_{\text{max}} - R_{\text{min}} )
\]

(5-25)

where,

\( R_{\text{max}} \): The maximum values of distances \( R_i \)
\( R_{\text{min}} \): The minimum values of distances \( R_i \) (Wen, et al., 2013).

Therefore, the parameters of the assessment cylinders are estimated by applying the following procedure.

STEP 1: Setting of parameters \( l_0 \) and \( m_0 \)

The parameters \( l_0 \) and \( m_0 \) are set in the initial values -0.010 and moved to +0.010 by the step of 0.001.

STEP 2: Estimation of parameters \( a_0, b_0, \) and \( R_0 \)

The parameters \( a_0, b_0, \) and \( R_0 \) are estimated by applying the Eqs. (5-21) to (-5-23) for all the combination of \( l_0 \) and \( m_0 \).

STEP 3: Estimation of distances from the measured \( R_i \) by applying the Eq. (5-24) and select an optimal parameters

The minimum zone cylindricity errors \( t \) are calculated by applying Eq. (5-25) based on the parameters of \( l_0, m_0, a_0, b_0 \) and the generated points \( P_i( X_i, Y_i, Z_i ) \). A combination of the parameter values which minimize \( t \) is selected as the parameters representing the assessment cylinder. The
parameters \( l_0, m_0, a_0 \) and \( b_0 \) obtained here give the position and orientation of the assessment cylinders.

### 5.6.2 Case study

In this section shows an example of the generated faces by boring and turning process simulations considering kinematic deviations. As shown in the Fig. 3-16 to 3-19, a set of the points on the bored and turned faces are generated. The geometric deviations of any points on the bored and turned faces can be evaluated by comparing the generated faces with the kinematic deviations. In order to validate the virtual machining module of boring and turning processes, two parts are considered between the bored and turned faces without kinematic deviations and another one with kinematic deviations.

#### 5.6.2.1 Estimation of bored and turned faces without kinematic deviations

Figure 3-16 and 3-17 show the simulation of the virtual machining of the bored and turned faces without kinematic deviations, which is simulated by program MATLAB in order to show the discrepancy of the machined and nominal surfaces. The simulations of the boring and turning processes are carried out based on Eq. (3-35) and (3-36). The machining parameters of boring and turning processes are same as ones for the simulations without kinematic motion deviations shown in the previous chapter 3.

**Table 5-7 Estimated 3D tolerances of boring and turning process simulations without kinematic deviations**

<table>
<thead>
<tr>
<th>Test no.</th>
<th>Cutting Condition</th>
<th>Surface Roughness</th>
<th>3-Dimensional Tolerances</th>
</tr>
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<tr>
<td></td>
<td>Nominal Diameter</td>
<td>( f )</td>
<td>( r_e )</td>
</tr>
<tr>
<td></td>
<td>[mm]</td>
<td>[mm/rev]</td>
<td>[mm]</td>
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<td>Test no.</td>
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</tr>
<tr>
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<td>Nominal Diameter</td>
<td>f [mm/rev]</td>
<td>r_e [mm]</td>
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<tr>
<td>12</td>
<td>40.0</td>
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Table 5-7 summarizes the 2D surface roughness \( R_z \), the 3D surface roughness \( S_z \) and the cylindricity obtained from the boring and turning process simulations under the various conditions of the feed rate \( f \) and the nose radius of the cutting edges \( r_e \). The simulation results are almost same as the theoretical 2D surface roughness \( R_{z\_th} \), and this shows that the proposed simulation is applicable to verification of the surface roughness and the cylindricity of the machining processes with kinematic motion deviations. The 2D and 3D surface roughness is evaluated by measuring the lines of 12.5 mm length and the areas 12.5 mm x 12.5 mm square referring to JIS B 0633:2001.

5.6.2.2 Estimation of bored and turned faces with kinematic deviations

Figure 3-18 and 3-19 show the simulation of the virtual machining of the bored and turned faces with kinematic deviations. The machining parameters are same as ones for the simulations without kinematic motion deviations shown in the previous chapter 3.

Tables 5-8 and 5-9 summarize the 2D surface roughness \( R_z \), the 3D surface roughness \( S_z \) and the cylindricity obtained from the boring and turning process simulations under the various conditions of the feed rate \( f \) and the nose radius of the cutting edges \( r_e \). The obtained surface roughness and the cylindricity are much greater than the theoretical values and also the simulation results shown in Table 5-7 due to the kinematic motion deviations.

The cylindricity is larger than the surface roughness, since the cylindricity is evaluated for all the areas of the generated surfaces. The 2D surface roughness is verified along one line and the 3D surface roughness is verified with in an area. Therefore, the 2D surface roughness is smaller than the 3D ones. The lines and the areas for evaluation of the surface roughness are selected randomly from the generated surfaces of the simulations, and the obtained values are averaged.
Table 5-8 Estimated surface roughness and cylindricity in boring process simulations with kinematic motion deviations

<table>
<thead>
<tr>
<th>Test no.</th>
<th>Cutting Conditions</th>
<th>Surface Roughness</th>
<th>3-Dimensional Tolerances</th>
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<td>$r_e$ [mm]</td>
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Table 5-9 Estimated surface roughness and cylindricity in turning process simulations with kinematic motion deviations

<table>
<thead>
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</table>

The evaluated surface roughness and the cylindricity are not so affected by the feed rate $f$ and the nose radius $r_e$ as the case of the simulation without kinematic motion deviations. This means that effects of the kinematic motion deviations are greater than ones of the machining parameters.

### 5.7 Conclusion

A mathematical model has been proposed to represent the kinematic motion deviations of rotary tables and the five-axis machining centers. The proposed model has been applied to the analysis of kinematic motion deviations of two-axis rotary tables and the five-axis machining centers. The proposed model was also used to estimate the 3D tolerances in boring and turning processes based on simulation of virtual machining including kinematic motion deviations. The following conclusions are drawn:

1. A systematic model is proposed to represent the kinematic motion deviations of the machine tools, based on the geometric tolerances of the guide-ways of the rotary tables and the linear tables consisting the five-axis machining centers.

2. The proposed geometric deviation model of the rotary tables and the five-axis machine tools are applied to estimation of the geometric deviations of the rotary tables, considering the priorities among the guide-ways and the bearings of the rotary tables. The kinematic motion deviations of the five-axis machine tools are also estimated by applying the proposed model.

3. The proposed method provides us with a systematic method to analyze and to evaluate the kinematic motion deviations of the rotary tables and the five-axis machine tools, based on both the mean values and the standard deviations of the points. It is understood that the kinematic motion deviations are affected by the geometric tolerances of the guide-ways and
that the geometric tolerances of the guide-ways of rotary tables have higher effects on the kinematic motion deviations than the ones of linear tables.

(4) A model is proposed to represent the kinematic motions of the cutting edges against the workpieces, taking into consideration of the kinematic deviations of the milling machines and the turning machines. The proposed model is applied to the simulation of the simple boring and turning processes.

(5) A systematic method is proposed to estimate the cylindricity of the bored and turned faces based on the kinematic motions of the cutting edges. The geometries of the bored and turned faces are estimated, based on the cutting conditions, the tool geometries and the kinematic deviations of the boring and turning processes.

(6) Some case studies have been carried out to estimate the 3-dimensional tolerances based on both the boring and turning process simulations with the kinematic motion deviations. It is found, through case studies, that the proposed method provides us with a systematic method to verify and to evaluate the geometric dimensioning and tolerancing based on the minimum zone cylinders.
Chapter 6

Conclusions
6.1 Conclusions

The dissertation is composed of six chapters, and the outlines for each individual chapter are as follows.

Chapter 1 provides a brief introduction of the historical background of the research, and clarifies the necessity and objective of the research.

Chapter 2 reviews the background and importance of the conventional methods and new approaches for analysis kinematic motion deviations and the geometric deviations, virtual machining, 3D surface roughness and 3D tolerances based on the literature survey, new areas for research are identified and proposed.

Chapter 3 discusses the virtual machining model representing the virtual machining centers for the boring processes and virtual turning centers for the turning processes, which include the kinematic motion deviations. The bored and turned faces generated are obtained as a set of points and evaluated to investigate the effect of various process parameters on the geometry of the parts. Once the part is verified virtually, the performance of the machining process parameters to generate a part that meets the required quality specifications can be investigated for estimation the 3D surface roughness and tolerances based on the simulation results. The following items are concluded.

(1) A model is proposed to represent the kinematic motions of the cutting edges against the workpieces fixed on the spindles, taking into consideration of the kinematic deviations of the boring tool systems of the machining centers and turning tool systems of the turning centers. The model proposed here represent the positions of the cutting edges against the workpieces by applying the 4 by 4 transformation matrices including the kinematic motion deviations.

(2) A systematic method is proposed to estimate the geometric deviations of the machined face based on the kinematic motions of the cutting edges. A set of points on the bored and the turned faces are generated by applying the proposed method and the characteristics features of the generated faces are estimated based on the points.

(3) A proposed model and method are applied to the simulation of the simple boring and turning processes. The geometries of the bored and turned faces are estimated, based on the cutting conditions, the tool geometries and the kinematic deviations of the boring and turning processes.

Chapter 4 discusses the estimation of 3D surface roughness of produced by virtual boring and turning machining processes. The estimation of 3D surface roughness estimated both the 2-dimensional (2D) and 3D through the boring and turning process simulations with kinematic motion deviations. A model is proposed to represent the kinematic motions of the cutting edges against the workpieces, taking into consideration the kinematic deviations of the machining
centers and the turning centers. Details of the virtual machining module are proposed to estimate the geometric deviations of the machined face based on the kinematic motions of the cutting edges. A proposed model is applied to the simulation of the simple boring and turning process, the geometries of the machined faces are estimated, based on the cutting conditions, the tool geometries and the kinematic deviations of the boring and turning processes. A method is also proposed to estimate both the 2D and 3D surface roughness based on the boring and turning process simulation with the kinematic motion deviations. Case studies are shown with examples of the generated faces by boring and turning process simulations considering kinematic deviations in order to estimate the 3D surface roughness. The following items are concluded.

(1) A model is proposed to represent the kinematic motions of the cutting edges against the workpieces, taking into consideration of the kinematic deviations of the milling machines and the turning machines.

(2) The proposed model was applied to the simulation of simple boring and turning processes. The geometries of the bored and turned faces were obtained, based on the cutting conditions, the tool geometries and the kinematic deviations of the boring and turning processes.

(3) A systematic method is proposed to estimate the 2D and 3D surface roughness of the bored and turned faces based on the kinematic motions of the cutting edges, based on both the ISO and JIS standards dealing with the surface roughness evaluations. The proposed method provides us with a systematic method to verify and to evaluate the surface roughness based on the least mean square cylinders.

(4) The proposed method was applied to the estimation of both the 2D and 3D surface roughness, based on the simulation results of both the boring and turning processes. In the cases of the simulation without kinematic motion deviations, the results were same as the theoretical values. The disturbances in the surface roughness were numerically evaluated in the simulation results with kinematic motion deviations.

Chapter 5 discusses the estimation of 3D tolerances including kinematic motion deviations. In the beginning, a mathematical model of kinematic motion deviations of machine tools is discussed on the basis of the geometric tolerances. The shape generation motions are bases for analysis of machine tools including both the linear tables and rotary tables. The 3D tolerances of boring and turning processes are analysed based on the machine tool models including kinematic motion deviations. A systematic method is proposed in this section to simulate the shape generation processes in both the boring and turning operations, to estimate the geometric dimensioning and tolerancing of both bored and turned faces, based on the machining parameters. The shape generation motions with deviations are mathematically described by combining 4 by 4 transformation matrices. A set of points on the bored and turned faces are generated through the simulations, and an assessment surface is obtained as the datum reference to estimate the 3D
tolerances, based on the points generated by the boring and turning process simulations. The following items are concluded.

(1) A systematic model is proposed to represent the kinematic motion deviations of the machine tools, based on the geometric tolerances of the guide-ways of the rotary tables and the linear tables consisting the five-axis machining centers.

(2) The proposed geometric deviation model of the rotary tables and the five-axis machine tools are applied to estimation of the geometric deviations of the rotary tables, considering the priorities among the guide-ways and the bearings of the rotary tables. The kinematic motion deviations of the five-axis machine tools are also estimated by applying the proposed model.

(3) The proposed method provides us with a systematic method to analyze and to evaluate the kinematic motion deviations of the rotary tables and the five-axis machine tools, based on both the mean values and the standard deviations of the points. It is understood that the kinematic motion deviations are affected by the geometric tolerances of the guide-ways and that the geometric tolerances of the guide-ways of rotary tables have higher effects on the kinematic motion deviations than the ones of linear tables.

(4) A model is proposed to represent the kinematic motions of the cutting edges against the workpieces, taking into consideration of the kinematic deviations of the milling machines and the turning machines. The proposed model is applied to the simulation of the simple boring and turning processes.

(5) A systematic method is proposed to estimate the cylindricity of the bored and turned faces based on the kinematic motions of the cutting edges. The geometries of the bored and turned faces are estimated, based on the cutting conditions, the tool geometries and the kinematic deviations of the boring and turning processes.

(6) Some case studies have been carried out to estimate the 3-dimensional tolerances based on both the boring and turning process simulations with the kinematic motion deviations. It is found, through case studies, that the proposed method provides us with a systematic method to verify and to evaluate the geometric dimensioning and tolerancing based on the minimum zone cylinders.
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<td>A Study on Estimation of Three-Dimensional Tolerances Based on Simulation of Virtual Machining in Boring and Turning Processes Including Kinematic Motion Deviations</td>
<td>W. Thasana, N. Sugimura, K. Iwamura, Y. Tanimizu</td>
<td>Journal of Advanced Mechanical Design, Systems, and Manufacturing, JSME (Submitted)</td>
<td>Chapter 5</td>
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