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A Note on Law and Economics
–Wage Rate Leadership, Price Leadership, Quality Leadership and Unemployment in the Absence of Efficiency Wages–

Shigeru Watanabe
Miki Maeda

Abstract: Even in the absence of the efficiency wages, the possibility of the unemployment is examined considering the wage rate leadership and the quality leadership in addition to the price leadership.

In the first case where the wage rate leadership and the price leadership are considered, the optimal ratio between the elasticity of the leader’s output with respect to the wage rate and that with respect to the price will be equal to the ratio between the total wage payment and proceeds. This optimal condition does not be affected by the possible excess supply of the labor, i.e. the unemployment, since the wage rate is determined by the optimal wage rate leadership. In the same way, in the second case where the quality leadership is also considered in addition to the wage rate leadership and the price leadership the similar results have been obtained. These optimal conditions are not affected by the possible excess supply of labor; hence, the possibility of the unemployment cannot be excluded even in both cases. Hence, from the analysis of this note the following policy implication can also be obtained; in addition to the explicit collusion which violates the Antitrust [Antimonopoly] Law, the firms’ behaviors such as the price leadership, the wage rate leadership and the quality leadership must be adequately dealt with.

Keywords: Wage Rate Leadership, Price Leadership, Quality Leadership, Efficiency Wages, Unemployment, Antitrust [Antimonopoly] Law
1. Introduction

In Watanabe and Maeda (2013) the relationship between efficiency wages\(^1\) and wage rate leadership instead of price leadership\(^2\) has been analyzed. A purpose of this note is to show the possibility of unemployment even in the absence of efficiency wages, when both the wage rate leadership and the price leadership are taken into consideration. When the quality leadership is also considered in addition to the wage rate leadership and the price leadership, the similar results can also be obtained. In the next section the wage rate leadership and the price leadership will be examined. In section 3 the quality leadership will also be examined in addition to the wage rate leadership and the price leadership. In the case where both the wage rate leadership and the price leadership are considered the optimal ratio between the elasticity of the leader’s output with respect to the wage rate and that with respect to the price will be equal to the ratio between the total wage payment and proceeds. This optimal condition does not be affected by the possible excess supply of the labor, i.e. the unemployment, wage rate is determined by the optimal wage rate leadership. In the same way, in the case where the quality leadership is also considered as well as the wage rate leadership and the price leadership the similar results have been obtained. In the last section concluding remarks will be given.

2. Wage Rate Leadership, Price Leadership and Unemployment

It is assumed in this section that the leader determines the wage rate, \(w\), and the price, \(p\). Hence the profit, \(\pi\), of the leader is denoted by the equation (1).

\[
\pi = px - wM(x) = p(D(p) - \sum_{i=1}^{n} X_i (p, w)) - wM(D(p) - \sum_{i=1}^{n} X_i (p, w)), \tag{1}
\]

where \(X\) is the output level of the leader, \(M\) is the amount of labor employed by the leader which depend on \(X\). \(D(p)\) is the demand of the market which depends on \(p\). \(x (p, w) (i = 1...n)\) is the output levels of the \(n\) small firms which depend on \(p\) and \(w\). then
\[ X = d(p) - \sum_{i=1}^{n} x_i(p, w) \] is the output level of the leader.

Differentiating \( \pi \) with respect to \( w \) and \( p \) yields the following first order conditions (2) and (3). Second order conditions are assumed to be satisfied.

\[
\frac{\partial \pi}{\partial w} = p \left( -\sum_{i=1}^{n} \frac{\partial x_i}{\partial w} \right) - M + w \frac{dM}{dX} \sum_{i=1}^{n} \frac{\partial x_i}{\partial w} = 0, \tag{2}
\]

\[
\frac{\partial \pi}{\partial p} = D(p) - \sum_{i=1}^{n} x_i(p, w) + p \left( \frac{dD}{dp} - \sum_{i=1}^{n} \frac{\partial x_i}{\partial p} \right) - w \frac{dM}{dX} \left( \frac{dD}{dp} - \sum_{i=1}^{n} \frac{\partial x_i}{\partial p} \right) = v. \tag{3}
\]

From (2) and (3) the following result (4) can be obtained straightforwardly.

\[
\frac{\eta^X_w}{\eta^X_p} = \frac{wM}{pX}. \tag{4}
\]

where \( \eta^X_w \) is the elasticity of the leader’s output with respect to the wage rate, \( \eta^X_p \) is the elasticity of the leader’s output with respect to the price, \( wM \) is the total amount of the wage and \( pX \) is the proceeds of the leader.

From the condition (4) the following result can be obtained. In this case where both the wage rate leadership and the price leadership are simultaneously considered the ratio between the elasticity of the leader’s output with respect to the wage rate and that with respect to the price will be equal to the ratio between the total wage payment and proceeds. Hence, if the wage rate and the price determined by the wage rate leadership and the price leadership respectively do not satisfy the condition (4), then the wage rate and the price will not be optimal. Further, this condition is not affected by the possible excess supply of labor, hence, the possibility of the unemployment cannot be excluded in this case.

3. Wage Rate Leadership, Quality Leadership, Price Leadership and Unemployment

In this section the quality leadership will also be examined in addition to the wage rate leadership and the price leadership.

The profit of the leader in this case is denoted by the following equation (5).
\[ \pi = pX - wM(X) \]
\[ = p(D(p, q) - \sum_{i=1}^{n} x_i(p, w, q)) - wM(D(p, q) - \sum_{i=1}^{n} x_i(p, w, q), q), \]

where \( q \) is the quality of the product determined by the leader.

Maximizing (5) with respect to \( w, p \) and \( q \) yields the following first order conditions.

Second order conditions are assumed to be satisfied.

\[ \frac{\partial \pi}{\partial w} = p \left( -\sum_{i=1}^{n} \frac{\partial x_i}{\partial w} \right) - M + w \frac{dM}{dx} \sum_{i=1}^{n} \frac{\partial x_i}{\partial w} = 0, \]  \hspace{1cm} (6)

\[ \frac{\partial \pi}{\partial p} = D(p, q) - \sum_{i=1}^{n} x_i(p, w, q) + p \left( \frac{dD}{dp} - \sum_{i=1}^{n} \frac{\partial x_i}{\partial p} \right) - w \frac{dM}{dx} \left( \frac{dD}{dp} - \sum_{i=1}^{n} \frac{\partial x_i}{\partial p} \right) = 0. \]  \hspace{1cm} (7)

\[ \frac{\partial \pi}{\partial q} = p \left( \frac{dD}{dq} - \sum_{i=1}^{n} \frac{\partial x_i}{\partial q} \right) - w \left( \frac{dM}{dx} \left( \frac{dD}{dq} - \sum_{i=1}^{n} \frac{\partial x_i}{\partial q} \right) + \frac{\partial M}{\partial q} \right) = 0. \]  \hspace{1cm} (8)

where \( \frac{\partial x}{\partial q} > 0 \) and \( \frac{\partial M}{\partial q} > 0 \) are assumed.

From (6), (7) and (8) the following results (9) and (10) can be obtained straightforwardly.

\[ \frac{\eta_w}{\eta_p} = \frac{wM}{pX}, \]  \hspace{1cm} (9)

\[ \frac{\eta_q}{\eta_q} = \frac{\eta_q}{\eta_q}, \]  \hspace{1cm} (10)

where \( \eta_q^M \) is the elasticity of the leader’s labor employment with respect to the quality of the product and \( \eta_q^X \) is the elasticity of the leader’s output with respect to the quality of the product.

Similar to (4), from (9) the ratio between the elasticity of the leader’s output with respect to the wage rate and that with respect to the price will be equal to the ratio between the total wage payment and proceeds. From (10) the additional condition is obtained such that the elasticity of the leader’s labor employment with respect to the quality of the product is equal to the ratio between the elasticity of the leader’s output
with respect to the quality and that with respect to the wage rate. Hence, if the wage rate, the price and the quality determined by the wage rate leadership, the price leadership and the quality leadership respectively do not satisfy the condition (9) and (10), then the wage rate, the price and the quality will not be optimal. Further, also in this case, these conditions are not affected by the possible excess supply of labor; hence, the possibility of the unemployment cannot be excluded even in this case.

4. Concluding Remarks

From the analysis of this note the following results have been obtained. Even in the absence of the efficiency wages, the possibility of the unemployment can not be excluded when the wage rate leadership and the quality leadership are considered in addition to the price leadership.

In the first case where the wage rate leadership and the price leadership are considered the ratio between the elasticity of the leader’s output with respect to the wage rate and that with respect to the price will be equal to the ratio between the total wage payment and proceeds. This optimal condition does not be affected by the possible excess supply of the labor, the possibility of the unemployment can not be excluded. In the second case where the quality leadership is also considered in addition to the wage rate leadership and the price leadership, the similar result and the additional result have been obtained such that the elasticity of the leader’s labor employment with respect to the quality of the product is equal to the ratio between the elasticity of the leader’s output with respect to the quality and that with respect to the wage rate. Also in this second case, these conditions are not affected by the possible excess supply of labor; hence, the possibility of the unemployment cannot be excluded even in this second case.

Hence, from the analysis of this note the following policy implication can also be obtained; in addition to the explicit collusion which violates the Antitrust [Antimonopoly] Law, the firms’ behaviors such as the price leadership, the wage rate leadership and the quality leadership must be adequately dealt with.

2 See Lynch (1946), for the price leadership.
References