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X-inefficiency and Y-inefficiency

Shigeru Watanabe*

Abstract

The first purpose of this paper is to analyze the relationship between $X$-inefficiency and tax evasion. The second purpose of this paper is to analyze the relationship between Pareto optimal redistribution and $Y$-inefficiency. The $Y$-inefficiency is related to utility function instead of cost function. In the presence of the $Y$-inefficiency, in order to attain the given level of the utility, the income larger than the minimum income which is required to attain that utility level is needed. Some parts of the consumption goods bought by the income will not be consumed but will be thrown away. If the strict consumption planning is made to attain the given utility level, the consumption goods which will be thrown aside as waste matter will vanish and only the minimum income will be needed to attain that utility level. However, the strict consumption planning needs strenuous efforts.

Following results have been obtained. (i) As is well known [See Watanabe (1988)], when the tax evasion is considered raising the tax rate will increase the output level of the monopoly in general. However, in the presence of the $X$-inefficiency a simple model can be shown where raising the tax rate will decrease the output level of the monopoly but increase the $X$-inefficiency. Hence even in the presence of the tax evasion the effect of raising the tax rate on the output level of the monopoly cannot be determined in general. (ii) In the presence of $Y$-inefficiency, an increase in the altruistic mind relative to the selfish mind will increase not only the amount of the Pareto optimal redistribution but also the rate of the consumption goods which will be thrown away as leftover food and drink i.e. as waste matter. Hence, the trade off exists between the voluntary private transfer of the Pareto optimal redistribution and $Y$-inefficiency.

Keywords: $X$-inefficiency, tax evasion, tax rate, monopoly, output level, $Y$-inefficiency, Pareto optimal redistribution, leftover food and drink

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1 Introduction

The first purpose of this paper is to analyze the relationship between $X$-inefficiency and tax evasion.

The second purpose of this paper is to analyze the relationship between Pareto optimal redistribution and $Y$-inefficiency. The $Y$-inefficiency is related to utility function instead of cost function. In the presence of the $Y$-inefficiency, in order to attain the given level of the utility, the income larger than the minimum income which required to attain that utility level is needed.

Some parts of the consumption goods bought by the income will not be consumed but will be thrown away as leftover food and drink i.e. as waste matter. If the strict consumption planning is made to attain the given utility level, the consumption goods which will be thrown aside as waste matter will vanish and only the minimum income will be needed to attain that utility level. However, the strict consumption planning needs strenuous efforts. Following results have been obtained.

(i) As is well known [See Watanabe (1988)], when the tax evasion is taken into consideration, raising the tax rate will increase the output level of the monopoly in general. However, in the presence of the $X$-inefficiency a simple model can be shown where raising the tax rate will decrease the output level of the monopoly but increase the $X$-inefficiency.

Hence, even in the presence of the tax evasion the effect of raising the tax rate on the output level of the monopoly cannot be determined in general.

(ii) In the presence of $Y$-inefficiency, an increase in the altruistic mind relatively to the selfish mind will increase not only the amount of the Pareto optimal redistribution but also the rate of the consumption goods which will be thrown away as waste matter.

In the next section the tax evasion of the monopoly in the presence of $X$-inefficiency will be examined. In section 3 the relationship between Pareto optimal redistribution and $Y$-inefficiency will be analyzed. In the last section concluding remarks will be given.

2 Tax Evasion of Monopoly in the Presence of $X$-inefficiency

The expected utility $EU$ of the monopolist is assumed to be denoted by the following equation.

$$EU = \alpha \log Q \left\{ (1 - t) (P(Q) - r(X)c_b) + (1 - q(\delta)F_t \delta r(X)c_b) \right\}$$

$$+ \beta X,$$

where $Q$ is the output level, $P(Q)$ is the price of the output, $t$ is the tax rate, $c$ is the constant average
cost, \( r(X) \) is an increasing function of \( X \)-inefficiency and denotes an increase in the average cost, \( \delta \) is the cost overstatement rate of tax evasion, \( q(\delta) \) is the probability of being detected and is an increasing function of \( \delta \), \( F \) is the penalty rate of the tax evasion, \( \alpha \) is weight of selfish mind, and \( \beta \) is the weight of the utility from the \( X \)-inefficiency.

Maximizing the equation (1) with respect to \( X, Q \) and \( \delta \) yields the following first order conditions.

\[
\frac{\partial EU}{\partial X} = \frac{\alpha - (1-t)r'(X)c_0 + (1-q(\delta)F)t\delta r'(X)c_0}{(1-t)(P(Q)-r(X)c_0)+(1-q(\delta)F)t\delta r(X)c_0} + \beta = 0, \\
\frac{\partial EU}{\partial Q} = \frac{\alpha (1-t)(P(Q)-r(X)c_0)+(1-q(\delta)F)t\delta r(X)c_0+Q(1-t)P'(Q)}{((1-t)(P(Q)-r(X)c_0)+(1-q(\delta)F)t\delta r(X)c_0)Q} = 0, \\
\frac{\partial EU}{\partial \delta} = \frac{\alpha - q'Ft\delta r(X)c_0 + (1-q(\delta)F)tr(X)c_0}{(1-t)(P(Q)-r(X)c_0)+(1-q(\delta)F)t\delta r(X)c_0} = 0.
\]

(2)

(3)

(4)

Second order conditions are assumed to be satisfied.

From the first order conditions the optimal \( X^*, Q^* \) and \( \delta^* \) can be obtained in the following manner.

\( P(Q), q(\delta) \) and \( r(x) \) are specified such that \( P(Q) = a - bQ, q(\delta) = \delta \) and \( r(x) = 1 + r_0x. \)

\[
X^* = \frac{(1-t)(\alpha - c_0) + \frac{c_0t}{4F}}{r_0c_0(1-t-\frac{t}{4F})} - \frac{2\alpha}{\beta}. \\
Q^* = \frac{\alpha r_0c_0}{b\beta} \left( 1 - \frac{t}{4(1-t)F} \right). \\
\delta^* = \frac{1}{2F}.
\]

(5)

(6)

(7)

The effect of raising the tax rate on \( X^*, Q^* \) and \( \delta^* \) can be obtained straightforwardly.
\[
\frac{\partial X^*}{\partial t} = \frac{c_0 \alpha}{4r_0c_0(1 - t - \frac{t}{4F})^2F} > 0, \quad (8)
\]

\[
\frac{\partial Q^*}{\partial t} = \frac{-\alpha r_0c_0}{4b \beta (1 - t)^2F} < 0. \quad (9)
\]

\[
\frac{\partial \delta^*}{\partial t} = 0. \quad (10)
\]

In the same way the effect of raising the penalty rate of the tax evasion on \(X^*\), \(Q^*\) and \(\delta^*\) can be obtained straightforwardly.

\[
\frac{\partial X^*}{\partial F} = \frac{-\alpha (1 - t)t}{4c_0r_0(1 - t - \frac{t}{4F})^2F^2} < 0, \quad (11)
\]

\[
\frac{\partial Q^*}{\partial F} = \frac{\alpha r_0c_0t}{4b \beta (1 - t)F^2} > 0, \quad (12)
\]

\[
\frac{\partial \delta^*}{\partial F} = \frac{-1}{2F^{-2}} < 0. \quad (13)
\]

Hence, the following results have been derived. In this simple model raising the tax rate will decrease the output level of the monopolist from (9) but will increase the \(X\)-inefficiency from (8). \(\delta\) will not be changed from (10). However, from (12) and (11) raising the penalty rate of the tax evasion will increase the output level of the monopolist but will decrease the \(X\)-inefficiency. And from (13) \(\delta\) will be decreased by the increase in the penalty rate of the tax evasion. As is well known, when the tax evasion is considered raising the tax rate will increase the output level of the monopoly in general. However, in the presence of the \(X\)-inefficiency a simple model has been shown where raising the tax rate will decrease the output level of the monopoly but increase the \(X\)-inefficiency. Hence, even in the presence of the tax evasion the effect of raising the tax rate on the output level of the monopoly cannot be determined in general.
3 Pareto Optimal Redistribution in the Presence of Y-inefficiency

In this section the relationship between Pareto optimal redistribution and Y-inefficiency will be examined.

The utility of the rich is denoted by the following equation (14).

\[
U = \theta \log \lambda(Y) (A - T) + \mu Y \\
+ \varepsilon \log (B + T),
\]

where \( \theta \) is the degree of selfish mind, \( \mu \) is the weight of the utility from Y-inefficiency, \( \varepsilon \) is the degree of altruistic mind, \( A \) is the initial income of the rich, \( B \) is the initial income of the poor, \( T \) is the amount of redistribution by the rich to the poor, \( 1 - \lambda \) is the rate at which the consumption goods are thrown away as waste matter, \( 1 - \lambda \) is assumed to be the increasing function of Y-inefficiency and the price of the consumption goods is normalized to be 1.

Maximizing the equation (14) with respect to \( T \) and \( Y \) yields the following first order conditions.

\[
\frac{\partial U}{\partial T} = \theta \frac{-1}{A - T} + \varepsilon \frac{1}{B + T} \\
= 0,
\]

\[
\frac{\partial U}{\partial Y} = \theta \frac{\lambda'(Y)}{\lambda(Y)} + \mu \\
= 0.
\]

Second order conditions are satisfied in the following manner.

\[
\frac{\partial^2 U}{\partial T^2} = -\theta (A - T)^{-2} - \varepsilon (B + T)^{-2} < 0.
\]
\[
\begin{vmatrix}
\frac{\partial^2 U}{\partial T^2} & \frac{\partial^2 U}{\partial T \partial Y} \\
\frac{\partial^2 U}{\partial Y \partial T} & \frac{\partial^2 U}{\partial Y^2}
\end{vmatrix}
= - \{ \theta (A - T)^{-2} + \varepsilon (B + T)^{-2} \} \theta \frac{\lambda''(Y)\lambda(Y) - \lambda'(Y)^2}{\lambda^2(Y)}
\]

< 0, \quad (18)

where \( \lambda^*(Y) \geq 0 \) is assumed.

From the first order conditions the optimal \( T^* \) and \( Y^* \) can be obtained in the following manner.

\[
T^* = \frac{\varepsilon A - \theta B}{\theta + \varepsilon}. \quad (19)
\]

\[
Y^* = \frac{1}{\lambda_0} - \frac{\theta}{\mu}, \quad (20)
\]

where \( \lambda(Y) \) is specified such that \( \lambda(Y) = 1 - \lambda_0 Y \).

The effects of the change in \( \theta, \mu \) and \( \varepsilon \) on \( T^* \) and \( Y^* \) can be obtained straightforwardly in the following manner.

\[
\partial T^*/\partial \theta = \frac{-\varepsilon (A + B)}{(\theta + \varepsilon)^2} < 0, \quad (21)
\]

\[
\partial Y^*/\partial \theta = - \frac{1}{\mu} < 0, \quad (22)
\]

\[
\partial T^*/\partial \mu = 0, \quad (23)
\]

\[
\partial Y^*/\partial \mu = \theta \mu^{-2} > 0, \quad (24)
\]

\[
\partial T^*/\partial \varepsilon = \frac{\theta (A + B)}{(\theta + \varepsilon)^2} > 0, \quad (25)
\]

\[
\partial Y^*/\partial \varepsilon = 0. \quad (26)
\]
When \( \theta \) is decreased, while other parameters including \( \varepsilon \) are kept constant, the altruistic mind increases relatively to the selfish mind. Then from (21) and (22), if \( \theta \) is decreased, not only the amount of the Pareto optimal redistribution but also the \( Y \)-inefficiency will increase. Hence, \( \lambda \) will be decreased. Then \( 1 - \lambda \), (i.e. the rate at which the consumption goods will be thrown away as waste matter) will increase. Therefore, when the altruistic mind increases relatively to the selfish mind owing to the decrease in \( \theta \) without the change in \( \varepsilon \), there exists the trade off between the voluntary private transfer of the Pareto optimal redistribution and \( Y \)-inefficiency.

From (23), (24), (25) and (26) the following results can also be obtained. The effect of the increase in \( \mu \) will increase \( Y \)-inefficiency but has no effect on the amount of the Pareto optimal redistribution. The effect of the increase in \( \varepsilon \) will increase the amount of the Pareto optimal redistribution but has no effect on \( Y \)-inefficiency. Therefore, when the altruistic mind increases relatively to the selfish mind owing to the increase in \( \varepsilon \) without the change in \( \theta \) the trade off between the voluntary private transfer of the Pareto optimal redistribution and \( Y \)-inefficiency does not exist.

However, as is shown in the following, if the rate between the real consumption by the rich and that by the poor is taken into consideration the trade off exists even if the altruistic mind increases relatively to the selfish mind owing to the increase in \( \varepsilon \) without the change in \( \theta \).

When the rate between the real consumption by the rich and that by the poor is taken into consideration, the following utility function can be analyzed.

\[
U = \theta \log \lambda (Y) (A - T) + \mu Y + \varepsilon \log \frac{B + T}{\lambda (Y) (A - T)}.
\]  

Similarly, the following optimal value of \( T^{**} \) and \( Y^{**} \) in this case can also be obtained from the first order conditions which maximizes (27) with respect to \( T \) and \( Y \).

\[
T^{**} = \frac{\varepsilon (A - B)}{\theta} - B,
\]  

\[
Y^{**} = \frac{\mu + \varepsilon \lambda_0 - \lambda_0 \theta}{\mu \lambda_0}.
\]

The effects of the change in \( \theta \), \( \mu \), \( \varepsilon \) on \( T^{**} \) and \( Y^{**} \) can be obtained straightforwardly in the following manner.

\[
\partial T^{**}/\partial \theta = -\varepsilon (A + B) \theta^{-2} < 0,
\]

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$\partial Y^{**}/\partial \theta = -\frac{1}{\mu} < 0. \tag{31}$

$\partial T^{**}/\partial \mu = 0. \tag{32}$

$\partial Y^{**}/\partial \mu = \frac{(\theta - \epsilon) \lambda_0^2}{\mu^2 \lambda_0^2} > 0. \tag{33}$

where $\theta > \epsilon$ is assumed.

$\partial T^{**}/\partial \epsilon = \frac{A + B}{\theta} > 0. \tag{34}$

$\partial Y^{**}/\partial \epsilon = -\frac{1}{\mu} > 0. \tag{35}$

Hence following results can be obtained from (30), (31), (32), (33), (34) and (35). The increase in $\theta$ decreases both the amount of Pareto optimal redistribution and $Y$-inefficiency. The increases in $\mu$ increases $Y$-inefficiency but has no effect on the amount of Pareto optimal redistribution. The increase in $\epsilon$ increases both the amount of Pareto optimal redistribution and $Y$-inefficiency. Hence, the trade off exists between the voluntary private transfer of the Pareto optimal redistribution and $Y$-inefficiency regardless of the causes. Therefore, an increase in the altruistic mind relative to the selfish mind will increase not only the amount of the Pareto optimal redistribution but also the rate of the consumption goods which will be thrown away as leftover food and drink i.e. as waste matter.

4 Concluding Remarks

The relationship between $X$-inefficiency and tax evasion of the monopoly has been analyzed. As is well known [See Watanabe (1988)], when the tax evasion is considered raising the tax rate will increase the output level of the monopoly in general. However, in the presence of the $X$-inefficiency a simple model can be shown where raising the tax rate will decrease the output level of the monopoly but will increase the $X$-inefficiency. Hence even in the presence of the tax evasion the effect of raising the tax rate on the output level of the monopoly cannot be determined in general.

The relationship between Pareto optimal redistribution and $Y$-inefficiency has been analyzed. The $Y$-inefficiency is related to utility function instead of cost function. In the presence of the
$Y$-inefficiency in order to attain the given level of the utility, the income larger than the minimum income which required to attain that utility level is needed. Some parts of the consumption goods bought by the income will not be consumed but will be thrown away as waste matter. If the strict consumption planning is made to attain the given utility level, the consumption goods which will be thrown aside as waste matter will vanish and only the minimum income will be needed to attain that utility level. However, the strict consumption planning needs strenuous efforts.

If the altruistic mind increases relatively to the selfish mind, not only the amount of the Pareto optimal redistribution but also the $Y$-inefficiency will increase. Hence, the relative increase in altruistic mind will increase the rate at which the consumption goods will be thrown away will be thrown away as leftover food and drink i.e. as waste matter. Hence, the trade off exists between the voluntary private transfer of the Pareto optimal redistribution and $Y$-inefficiency.

Notes
1 See Leibenstein (1966) for $X$-inefficiency.

References
X - inefficiency and Y - inefficiency

———, “A Note on Taxation & Understated Monopoly Profit : Tax Evasion & Oversupply of Monopoly”