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<th>消費者のブランド選択活動に関する研究：ジュエリーやアクセサリー購入における消費者行動の分析</th>
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<tr>
<td>Author(s)</td>
<td>佐屋賀、和弘；石生、千恵</td>
</tr>
<tr>
<td>Editor(s)</td>
<td>大阪府立大学経済研究</td>
</tr>
<tr>
<td>Citation</td>
<td>大阪府立大学経済研究 58(1)，p.49-74</td>
</tr>
<tr>
<td>Issue Date</td>
<td>2012-06-20</td>
</tr>
<tr>
<td>URL</td>
<td><a href="http://hdl.handle.net/10466/12712">http://hdl.handle.net/10466/12712</a></td>
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<tr>
<td>Rights</td>
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Consumers’ Activities for Brand Selection in the Case of Jewelry/Accessory Purchasing

Kazuhiro Takeyasu · Chie Ishio

Abstract: It is often observed that consumers select upper class brand when they buy next time. Suppose that former buying data and current buying data are gathered. Also suppose that upper brand is located upper in the variable array. Then transition matrix becomes upper triangular matrix under the supposition that former buying variables are set input and current buying variables are set output. There may be also the case that customers select lower brand to seek suitable price when they have chosen higher brand. Then it may compose items of lower triangular matrix. Utilizing jewelry/accessory purchasing history record of on-line shopping over three years, above structure is investigated and confirmed. Comparison with our previous research is also executed. Some interesting results are obtained. Unless planner for products does not notice its brand position whether it is upper or lower than other products, matrix structure makes it possible to identify those by calculating consumers’ activities for brand selection. Thus, this proposed approach enables to make effective marketing plan and/or establishing new brand.

Key Words: brand selection, matrix structure, brand position, jewelry, accessory

1. INTRODUCTION

It is often observed that consumers select upper class brand when they buy next time. Suppose that former buying data and current buying data are gathered. Also suppose that upper brand is located upper in the variable array. Then transition matrix becomes upper triangular matrix under the supposition that former buying variables are set input and current buying variables are set output. If the top brand were selected from lower brand in jumping way, corresponding part in upper triangular matrix would be 0. These are verified in numerical examples with simple models. There may be also the case that customers select lower brand to seek suitable price when they have chosen higher brand. Then it may compose items of lower triangular matrix.
If transition matrix is identified, S-step forecasting can be executed. Generalized forecasting matrix components' equations are introduced. Unless planner for products does not notice its brand position whether it is upper or lower than other products, matrix structure makes it possible to identify those by calculating consumers' activities for brand selection. Thus, this proposed approach enables to make effective marketing plan and/or establishing new brand.


In this paper, matrix structure is analyzed for the case brand selection is executed for upper class and for lower class, utilizing jewelry/accessory purchasing history record of on-line shopping over three years. Comparison with our previous research (Takeyasu et al., 2011) is also executed. Some interesting results are obtained. Such research can not be found as long as searched.

Hereinafter, matrix structure is clarified for the selection of brand in section 2. Block matrix structure is analyzed when brands are handled in group and s-step forecasting is formulated in section 3. Purchase history investigation of jewelry/accessory on-line shopping is examined and its numerical calculation is executed in section 4. Application of this method is extended in section 5, which is followed by the remarks of section 6.

2. BRAND SELECTION AND ITS MATRIX STRUCTURE

(1) Upper shift of Brand selection

It is often observed that consumers select upper class brand when they buy next time. Now, suppose that \( x \) is the most upper class brand, \( y \) is the second upper brand, and \( z \) is the lowest brand. Consumer’s behavior of selecting brand would be \( z \rightarrow y \), \( y \rightarrow x \), \( z \rightarrow x \) etc. \( x \rightarrow z \) might be few.

Suppose that \( x \) is current buying variable, and \( x_b \) is previous buying variable. Shift to \( x \) is executed from \( x_b \), \( y_b \), or \( z_b \).

Therefore, \( x \) is stated in the following equation.

\[
x = a_{11}x_b + a_{12}y_b + a_{13}z_b
\]
Similarly,

$$y = a_{22} y_b + a_{23} z_b$$

and

$$z = a_{33} z_b$$

These are re-written as follows.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{pmatrix} \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix}$$

(1)

Set

$$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{pmatrix}$$

$$X_b = \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix}$$

then, $X$ is represented as follows.

$$X = AX_b$$

(2)

Here,

$$X \in \mathbb{R}^3, A \in \mathbb{R}^{3 \times 3}, X_b \in \mathbb{R}^3$$

$A$ is an upper triangular matrix.

To examine this, generating following data, which are all consisted by upper brand shift data,
\[
X^i = \begin{pmatrix}
1 \\
0 \\
0
\end{pmatrix}
\begin{pmatrix}
1 \\
0 \\
0
\end{pmatrix}
\cdots
\begin{pmatrix}
0 \\
1 \\
0
\end{pmatrix}
\]  
\tag{3}

\[
X^i_b = \begin{pmatrix}
0 \\
1 \\
0
\end{pmatrix}
\begin{pmatrix}
1 \\
0 \\
0
\end{pmatrix}
\cdots
\begin{pmatrix}
0 \\
0 \\
1
\end{pmatrix}
\]  
\tag{4}

\[i = 1, 2, \ldots, N\]

parameter can be estimated using least square method. Suppose

\[X^i = AX^i_b + \varepsilon^i\]  
\tag{5}

and

\[J = \sum_{i=1}^{N} \varepsilon^i \varepsilon^i \rightarrow Min\]  
\tag{6}

\[\hat{A}\text{ which is an estimated value of } A \text{ is obtained as follows.}\]

\[\hat{A} = \left( \sum_{i=1}^{N} X^i X^i_b \right) \left( \sum_{i=1}^{N} X^i_b X^i_b \right)^{-1}\]  
\tag{7}

In the data group of upper shift brand, estimated value \(\hat{A}\) should be upper triangular matrix. If following data that have lower shift brand are added only a few in equation (3) and (4),

\[
X^i = \begin{pmatrix}
0 \\
1 \\
0
\end{pmatrix}
\]

\[
X^i_b = \begin{pmatrix}
1 \\
0 \\
0
\end{pmatrix}
\]

\(\hat{A}\) would contain minute items in the lower part of triangle.
(2) Sorting brand ranking by re-arranging row

In a general data, variables may not be in order as $x, y, z$. In that case, large and small value lie scattered in $\hat{A}$. But re-arranging this, we can set in order by shifting row. The large value parts are gathered in upper triangular matrix, and the small value parts are gathered in lower triangular matrix.

$$
\hat{A} = \begin{pmatrix}
  \varepsilon & 0 & 0 \\
  \varepsilon & \varepsilon & 0 \\
  \varepsilon & \varepsilon & \varepsilon
\end{pmatrix}
$$

(3) In the case that brand selection shifts in jump

It is often observed that some consumers select the most upper class brand from the most lower class brand and skip selecting the middle class brand.

We suppose $v, w, x, y, z$ brands (suppose they are laid from upper position to lower position as $v > w > x > y > z$).

In the above case, selection shifts would be:

$$
v \leftarrow z
$$

$$
v \leftarrow y
$$

Suppose they do not shift to $y, x, w$ from $z$, to $x, w$ from $y$, and to $w$ from $x$. then Matrix structure would be as follows.

$$
\begin{pmatrix}
v \\
w \\
x \\
y \\
z
\end{pmatrix} = 
\begin{pmatrix}
a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\
0 & a_{22} & 0 & 0 & 0 \\
0 & 0 & a_{33} & 0 & 0 \\
0 & 0 & 0 & a_{44} & 0 \\
0 & 0 & 0 & 0 & a_{55}
\end{pmatrix}
\begin{pmatrix}
y_b \\
w_b \\
x_b \\
y_b \\
z_b
\end{pmatrix}
$$

(9)
3. BLOCK MATRIX STRUCTURE IN BRAND GOURPS AND \( \epsilon \)-STEP FORECASTING

Next, we examine the case in brand groups. Matrices are composed by Block Matrix.

(1) Brand shift group — in the case of two groups

Suppose brand selection shifts from Corolla class to Mark II class in car. In this case, it does not matter which company’s car they choose. Thus, selection of cars are executed in a group and brand shift is considered to be done from group to group. Suppose brand groups at time \( n \) are as follows.

\( \mathbf{X} \) consists of \( p \) varieties of goods, and \( \mathbf{Y} \) consists of \( q \) varieties of goods.

\[
\mathbf{X}_n = \begin{pmatrix}
  x_1^n \\
  x_2^n \\
  \vdots \\
  x_p^n 
\end{pmatrix}
\]

\[
\mathbf{Y}_n = \begin{pmatrix}
  y_1^n \\
  y_2^n \\
  \vdots \\
  y_q^n 
\end{pmatrix}
\]

\[
\begin{pmatrix}
  \mathbf{X}_n \\
  \mathbf{Y}_n 
\end{pmatrix} = \begin{pmatrix}
  \mathbf{A}_{11}, & \mathbf{A}_{12} \\
  0, & \mathbf{A}_{22}
\end{pmatrix} \begin{pmatrix}
  \mathbf{X}_{n-1} \\
  \mathbf{Y}_{n-1}
\end{pmatrix}
\]

Here,

\( \mathbf{X}_n \in \mathbb{R}^p \ (n = 1, 2, \cdots) \), \( \mathbf{Y}_n \in \mathbb{R}^q \ (n = 1, 2, \cdots) \), \( \mathbf{A}_{11} \in \mathbb{R}^{pp} \), \( \mathbf{A}_{12} \in \mathbb{R}^{pq} \), \( \mathbf{A}_{22} \in \mathbb{R}^{qq} \)

Make one more step of shift, then we obtain the following equation.

\[
\begin{pmatrix}
  \mathbf{X}_n \\
  \mathbf{Y}_n 
\end{pmatrix} = \begin{pmatrix}
  \mathbf{A}_{11}^2, & \mathbf{A}_{11} \mathbf{A}_{12} + \mathbf{A}_{12} \mathbf{A}_{22} \\
  0, & \mathbf{A}_{22}^2
\end{pmatrix} \begin{pmatrix}
  \mathbf{X}_{n-2} \\
  \mathbf{Y}_{n-2}
\end{pmatrix}
\]

Make one more step of shift again, then we obtain the following equation.
$$\begin{align*}
\begin{bmatrix} X_n \\ Y_n \end{bmatrix} &= \begin{bmatrix} A_{11}^2 + A_{11} A_{12}^2 + A_{12} A_{22}^2 \ A_{22}^3 \ 0 \end{bmatrix} \begin{bmatrix} X_{n-3} \\ Y_{n-3} \end{bmatrix} \\
\begin{bmatrix} X_n \\ Y_n \end{bmatrix} &= \begin{bmatrix} A_{11}^4 + A_{11}^3 A_{12} + A_{11}^2 A_{12} A_{22}^2 + A_{12} A_{22}^3 \ 0 \end{bmatrix} \begin{bmatrix} X_{n-4} \\ Y_{n-4} \end{bmatrix} \\
\begin{bmatrix} X_n \\ Y_n \end{bmatrix} &= \begin{bmatrix} A_{11}^5 + A_{11}^4 A_{12} + A_{11}^3 A_{12} A_{22} + A_{11}^2 A_{12}^2 A_{22}^2 + A_{11} A_{12} A_{22}^3 + A_{12} A_{22}^4 \ 0 \end{bmatrix} \begin{bmatrix} X_{n-5} \\ Y_{n-5} \end{bmatrix}
\end{align*}$$

Similarly,

$$\begin{align*}
\begin{bmatrix} X_n \\ Y_n \end{bmatrix} &= \begin{bmatrix} A_{11}^3 + A_{11} A_{12}^2 + A_{12} A_{22}^2 \ 0 \end{bmatrix} \begin{bmatrix} X_{n-3} \\ Y_{n-3} \end{bmatrix} \\
\begin{bmatrix} X_n \\ Y_n \end{bmatrix} &= \begin{bmatrix} A_{11}^4 + A_{11}^3 A_{12} + A_{11}^2 A_{12} A_{22}^2 + A_{12} A_{22}^3 \ 0 \end{bmatrix} \begin{bmatrix} X_{n-4} \\ Y_{n-4} \end{bmatrix} \\
\begin{bmatrix} X_n \\ Y_n \end{bmatrix} &= \begin{bmatrix} A_{11}^5 + A_{11}^4 A_{12} + A_{11}^3 A_{12} A_{22} + A_{11}^2 A_{12}^2 A_{22}^2 + A_{11} A_{12} A_{22}^3 + A_{12} A_{22}^4 \ 0 \end{bmatrix} \begin{bmatrix} X_{n-5} \\ Y_{n-5} \end{bmatrix}
\end{align*}$$

Finally, we get the generalized equation for $s$-step shift as follows.

$$\begin{align*}
\begin{bmatrix} X_n \\ Y_n \end{bmatrix} &= \begin{bmatrix} A_{11}^s + A_{11} A_{12}^2 + \sum_{k=2}^{s-1} A_{11}^{s-k} A_{12} A_{22}^{k-1} \ 0 \end{bmatrix} \begin{bmatrix} X_{n-s} \\ Y_{n-s} \end{bmatrix}
\end{align*}$$

If we replace $n - s \rightarrow n, n \rightarrow n + s$ in equation (15), we can make $s$-step forecast.

(2) Brand shift group— in the case of three groups

Suppose brand selection is executed in the same group or to the upper group, and also suppose that brand position is $x > y > z$ ($x$ is upper position). Then brand selection transition matrix would be expressed as

$$\begin{align*}
\begin{bmatrix} X_n \\ Y_n \\ Z_n \end{bmatrix} &= \begin{bmatrix} A_{11}, A_{12}, A_{13} \\ 0, A_{22}, A_{23} \\ 0, 0, A_{33} \end{bmatrix} \begin{bmatrix} X_{n-1} \\ Y_{n-1} \\ Z_{n-1} \end{bmatrix}
\end{align*}$$

Where

$$\begin{align*}
\begin{bmatrix} X_n \\
X''_1 \\
X''_2 \\
\vdots \\
X''_p \\
\end{bmatrix}
\end{align*}$$
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\[
Y_n = \begin{pmatrix}
  y_1^n \\
  y_2^n \\
  \vdots \\
  y_q^n
\end{pmatrix}
\]

\[
Z_n = \begin{pmatrix}
  z_1^n \\
  z_2^n \\
  \vdots \\
  z_r^n
\end{pmatrix}
\]

Here,

\[
X_n \in \mathbb{R}^f \ (n=1,2,\ldots), \quad Y_n \in \mathbb{R}^q \ (n=1,2,\ldots), \quad Z_n \in \mathbb{R}^r \ (n=1,2,\ldots), \quad A_{11} \in \mathbb{R}^{fp}, \quad A_{12} \in \mathbb{R}^{fq}
\]

\[
A_{13} \in \mathbb{R}^{fr}, \quad A_{22} \in \mathbb{R}^{qf}, \quad A_{23} \in \mathbb{R}^{qg}, \quad A_{33} \in \mathbb{R}^{rr}
\]

These are re-stated as

\[
W_n = AW_{n-1}
\]

(17)

Where

\[
W_n = \begin{pmatrix}
  X_n \\
  Y_n \\
  Z_n
\end{pmatrix}
\]

\[
A = \begin{pmatrix}
  A_{11}, & A_{12}, & A_{13} \\
  0, & A_{22}, & A_{23} \\
  0, & 0, & A_{33}
\end{pmatrix}
\]

\[
W_{n-1} = \begin{pmatrix}
  X_{n-1} \\
  Y_{n-1} \\
  Z_{n-1}
\end{pmatrix}
\]

Hereinafter, we shift steps as is done in the previous section.

In the general description, we state as
\[ W_n = A^{(s)}W_{n-s} \]  

Here,

\[
A^{(s)} = \begin{pmatrix}
A_{11}^{(s)}, & A_{12}^{(s)}, & A_{13}^{(s)} \\
0, & A_{22}^{(s)}, & A_{23}^{(s)} \\
0, & 0, & A_{33}^{(s)}
\end{pmatrix}
\]

\[
W_{n-s} = \begin{pmatrix}
X_{n-s} \\
Y_{n-s} \\
Z_{n-s}
\end{pmatrix}
\]

From definition,

\[ A^{(1)} = A \]  

In the case \( s = 2 \), we obtain

\[
A^{(2)} = \begin{pmatrix}
A_{11}, & A_{12}, & A_{13} \\
0, & A_{22}, & A_{23} \\
0, & 0, & A_{33}
\end{pmatrix} \begin{pmatrix}
A_{11}, & A_{12}, & A_{13} \\
0, & A_{22}, & A_{23} \\
0, & 0, & A_{33}
\end{pmatrix}
\]

\[
= \begin{pmatrix}
A_{11}, & A_{12}A_{12} + A_{13}A_{22}, & A_{11}A_{13} + A_{12}A_{23} + A_{13}A_{33} \\
0, & A_{22}, & A_{22}A_{23} + A_{23}A_{33} \\
0, & 0, & A_{33}
\end{pmatrix}
\]

Next, in the case \( s = 3 \), we obtain

\[
A^{(3)} = \begin{pmatrix}
A_{11}^2, & A_{11}^2A_{12} + A_{11}A_{12}A_{22} + A_{12}^2A_{22}, & A_{11}^2A_{13} + A_{11}A_{12}A_{23} + A_{12}^2A_{23} + A_{13}A_{33} + A_{13}A_{33} \\
0, & A_{22}^2, & A_{22}^2A_{23} + A_{23}A_{33} + A_{33}A_{33} \\
0, & 0, & A_{33}^2
\end{pmatrix}
\]

In the case \( s = 4 \), equations become wide-spread, so we express each Block Matrix as follows.
\[
A_{11}^{(4)} = A_{11}^4 \\
A_{12}^{(4)} = A_{11}^3 A_{12} + A_{11}^2 A_{12} A_{22} + A_{11} A_{12} A_{22}^2 + A_{12} A_{22}^3 \\
A_{13}^{(4)} = A_{11} A_{13} + A_{11} A_{12} A_{23} + A_{11}^2 A_{13} A_{33} + A_{11} A_{12} A_{22} A_{23} + A_{11} A_{12} A_{23} A_{33} + A_{11} A_{13} A_{33} + A_{12} A_{22} A_{23} + A_{12} A_{22} A_{23} A_{33} + A_{12} A_{23} A_{33} + A_{13} A_{33}^3 \\
A_{22}^{(4)} = A_{22}^4 \\
A_{23}^{(4)} = A_{22}^3 A_{23} + A_{22} A_{23} A_{33} + A_{22} A_{23} A_{33}^2 + A_{23} A_{33}^3 \\
A_{33}^{(4)} = A_{33}^4 \\
\]

In the case \( s = 5 \), we obtain the following equations similarly.

\[
A_{11}^{(5)} = A_{11}^5 \\
A_{12}^{(5)} = A_{11}^4 A_{12} + A_{11}^3 A_{12} A_{22} + A_{11}^2 A_{12} A_{22}^2 + A_{11} A_{12} A_{22}^3 + A_{12} A_{22}^4 \\
A_{13}^{(5)} = A_{11}^3 A_{13} + A_{11}^2 A_{13} A_{33} + A_{11} A_{13} A_{33}^2 + A_{11} A_{12} A_{22} A_{33} + A_{11} A_{12} A_{23} A_{33} + A_{11} A_{13} A_{33}^3 + A_{12} A_{22}^2 A_{23} + A_{12} A_{22} A_{23} A_{33} + A_{12} A_{23} A_{33} + A_{13} A_{33}^4 + A_{11} A_{13} A_{33}^3 + A_{12} A_{22} A_{23} + A_{12} A_{22} A_{23} A_{33} + A_{12} A_{23} A_{33} + A_{13} A_{33}^4 \\
A_{22}^{(5)} = A_{22}^5 \\
A_{23}^{(5)} = A_{22}^4 A_{23} + A_{22} A_{23} A_{33} + A_{22} A_{23} A_{33}^2 + A_{23} A_{33}^4 + A_{22} A_{23} A_{33}^3 + A_{23} A_{33}^4 \\
A_{33}^{(5)} = A_{33}^5 \\
\]

In the case \( s = 6 \), we obtain

\[
A_{11}^{(6)} = A_{11}^6 \\
A_{12}^{(6)} = A_{11}^5 A_{12} + A_{11}^4 A_{12} A_{22} + A_{11}^3 A_{12} A_{22}^2 + A_{11}^2 A_{12} A_{22}^3 + A_{11} A_{12} A_{22}^4 + A_{12} A_{22}^5 \\
A_{13}^{(6)} = A_{11}^4 A_{13} + A_{11}^3 A_{13} A_{33} + A_{11}^2 A_{13} A_{33}^2 + A_{11} A_{13} A_{33}^3 + A_{11} A_{12} A_{22} A_{33} + A_{11} A_{12} A_{23} A_{33} + A_{11} A_{13} A_{33}^4 + A_{12} A_{22}^2 A_{23} + A_{12} A_{22} A_{23} A_{33} + A_{12} A_{23} A_{33} + A_{13} A_{33}^5 + A_{11} A_{13} A_{33}^3 + A_{12} A_{22}^2 A_{23} + A_{12} A_{22} A_{23} A_{33} + A_{12} A_{23} A_{33} + A_{13} A_{33}^4 + A_{11} A_{13} A_{33}^3 + A_{12} A_{22} A_{23} + A_{12} A_{22} A_{23} A_{33} + A_{12} A_{23} A_{33} + A_{13} A_{33}^5 \\
\]

大阪府立大学経済研究 第58巻 1 (239) [2012.6]
We get generalized equations for $s$-step shift as follows.

$$
\begin{align*}
A_{11}^{(s)} &= A_{11}^s, \\
A_{12}^{(s)} &= A_{11}^{s-1}A_{12} + \sum_{k=2}^{s-1} A_{11}^{s-k}A_{22}A_{22}^{k-1} + A_{12}A_{22}^{s-1} \\
A_{13}^{(s)} &= A_{11}^{s-1}A_{13} + A_{11}^{s-2}\left( \sum_{k=1}^{s-1} A_{11}^{t(k-1)}A_{33}^{(k)} \right) + \sum_{j=1}^{s-1} A_{11}^{s-2-j}\left[ A_{12}\left( \sum_{k=1}^{j} A_{22}^{j-k}A_{23}A_{33}^{k-1} \right) + A_{13}A_{33}^{(j)} \right] \\
A_{22}^{(s)} &= A_{22}^s \\
A_{23}^{(s)} &= \sum_{k=1}^{s} A_{22}^{s-k}A_{23}A_{33}^{k-1} \\
A_{33}^{(s)} &= A_{33}^s
\end{align*}
$$

Expressing them in matrix, it follows.

$$
A^{(s)} = \begin{bmatrix}
A_{11}^{s-1}A_{12} + \sum_{k=2}^{s-1} A_{11}^{s-k}A_{22}A_{22}^{k-1} + A_{12}A_{22}^{s-1} \\
A_{11}^{s-2}\left( \sum_{k=1}^{s-1} A_{11}^{t(k-1)}A_{33}^{(k)} \right) + \sum_{j=1}^{s-1} A_{11}^{s-2-j}\left[ A_{12}\left( \sum_{k=1}^{j} A_{22}^{j-k}A_{23}A_{33}^{k-1} \right) + A_{13}A_{33}^{(j)} \right] \\
0 & A_{22}^s \\
0 & 0 & \sum_{k=1}^{s} A_{22}^{s-k}A_{23}A_{33}^{k-1} \\
0 & 0 & 0 & A_{33}^s
\end{bmatrix}
$$

Generalizing them to $m$ groups, they are expressed as

$$
\begin{bmatrix}
X_{n}^{(1)} \\
\vdots \\
X_{n}^{(m)}
\end{bmatrix} = 
\begin{bmatrix}
A_{11} & A_{12} & \cdots & A_{1m} \\
A_{21} & A_{22} & \cdots & A_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
A_{m1} & A_{m2} & \cdots & A_{mm}
\end{bmatrix}
\begin{bmatrix}
X_{n-1}^{(1)} \\
\vdots \\
X_{n-1}^{(m)}
\end{bmatrix}
$$

$$
X_{n}^{(1)} \in R^{k_1}, \ X_{n}^{(2)} \in R^{k_2}, \cdots, \ X_{n}^{(m)} \in R^{k_m}, \ A_{ij} \in R^{k_{i} \times k_{j}}, (i=1,\cdots,m)(j=1,\cdots,m)
$$
4. PURCHASE HISTORY INVESTIGATION and NUMERICAL CALCULATION

Jewelry/accessory purchase history investigation is executed.

First of all, the framework of jewelry/accessory purchasing via on-line shopping is as follows.

- On-line shop: Ciao! / Happy gift
  Host site: http://www.happy-gift.jp/
  Branch site: http://www.rakuten.co.jp/ciao/
  http://store.shopping.yahoo.co.jp/b-ciao/index.html

  Managed by Cherish Co. Ltd.

- Customers: all over Japan (Every Prefecture)
- Data gathering period: April 2008 – May 2011
- Order number: 4411 (limited to the order number which has repeated order)

- Main residents of customers
  Tokyo 11.9%
  Kanagawa 8.7%
  Osaka 6.0%
  Aichi 5.8%
  Chiba 5.7%
  Saitama 5.4%

  The share of Tokyo capital area consist of 31.7%.

- Sales goods:
  Necklace / Pendant
  Pierced earrings
  Ring
  Bracelet / Bangle
  Brooch
  Necktie Pin
  Miscellaneous (Package/Ribbon etc.)

- Classification of goods by price
<table>
<thead>
<tr>
<th>Rank</th>
<th>Price (Yen)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N6</td>
<td>40001~</td>
</tr>
<tr>
<td>N5</td>
<td>~40000</td>
</tr>
<tr>
<td>N4</td>
<td>~30000</td>
</tr>
<tr>
<td>N3</td>
<td>~20000</td>
</tr>
<tr>
<td>N2</td>
<td>~15000</td>
</tr>
<tr>
<td>N1</td>
<td>~10000</td>
</tr>
<tr>
<td>P6</td>
<td>24001~</td>
</tr>
<tr>
<td>P5</td>
<td>~24000</td>
</tr>
<tr>
<td>P4</td>
<td>~16000</td>
</tr>
<tr>
<td>P3</td>
<td>~10000</td>
</tr>
<tr>
<td>P2</td>
<td>~6000</td>
</tr>
<tr>
<td>P1</td>
<td>~2000</td>
</tr>
<tr>
<td>R6</td>
<td>40001~</td>
</tr>
<tr>
<td>R5</td>
<td>~40000</td>
</tr>
<tr>
<td>R4</td>
<td>~30000</td>
</tr>
<tr>
<td>R3</td>
<td>~20000</td>
</tr>
<tr>
<td>R2</td>
<td>~15000</td>
</tr>
<tr>
<td>R1</td>
<td>~10000</td>
</tr>
<tr>
<td>B6</td>
<td>40001~</td>
</tr>
<tr>
<td>B5</td>
<td>~40000</td>
</tr>
<tr>
<td>B4</td>
<td>~35000</td>
</tr>
<tr>
<td>B3</td>
<td>~30000</td>
</tr>
<tr>
<td>B2</td>
<td>~15000</td>
</tr>
<tr>
<td>B1</td>
<td>~10000</td>
</tr>
</tbody>
</table>

The purchase history data was the most for Necklace/ Pendant. Therefore we make focus on them.

① Number of shift from N1 position to N1 position : 537
② Number of shift from N1 position to N2 position : 100
③ Number of shift from N1 position to N3 position : 44
④ Number of shift from N1 position to N4 d position : 12
⑤ Number of shift from N1 position to N5 position : 0
⑥ Number of shift from N1 position to N6 position : 0
⑦ Number of shift from N2 position to N1 position : 105
⑧ Number of shift from N2 position to N2 position : 687
⑨ Number of shift from N2 position to N3 position : 44
⑩ Number of shift from N2 position to N4 position : 11
⑪ Number of shift from N2 position to N5 position : 3
⑫ Number of shift from N2 position to N6 position : 1
⑬ Number of shift from N3 position to N1 position : 33
⑭ Number of shift from N3 position to N2 position : 41
⑮ Number of shift from N3 position to N3 position : 341
⑯ Number of shift from N3 position to N4 position : 13
⑰ Number of shift from N3 position to N5 position : 0
⑱ Number of shift from N3 position to N6 position : 1
⑲ Number of shift from N4 position to N1 position : 12
⑳ Number of shift from N4 position to N2 position : 21
⑳ Number of shift from N4 position to N3 position : 15
⑳ Number of shift from N4 position to N4 position : 125
⑳ Number of shift from N4 position to N5 position : 4
⑳ Number of shift from N4 position to N6 position : 1
⑳ Number of shift from N5 position to N1 position : 1
⑳ Number of shift from N5 position to N2 position : 5
⑳ Number of shift from N5 position to N3 position : 1
⑳ Number of shift from N5 position to N4 position : 2
⑳ Number of shift from N5 position to N5 position : 12
⑳ Number of shift from N5 position to N6 position : 0
⑳ Number of shift from N6 position to N1 position : 2
⑳ Number of shift from N6 position to N2 position : 1
⑳ Number of shift from N6 position to N3 position : 2
⑳ Number of shift from N6 position to N4 position : 2
⑳ Number of shift from N6 position to N5 position : 0
Number of shift from N6 position to N6 position : 2

Total: 2181

By the way, the total number of shifts in one genre is as follows.

<table>
<thead>
<tr>
<th>Genre</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Necklace / Pendant</td>
<td>2181</td>
</tr>
<tr>
<td>Pierced earrings</td>
<td>567</td>
</tr>
<tr>
<td>Ring</td>
<td>1205</td>
</tr>
<tr>
<td>Bracelet / Bungle</td>
<td>458</td>
</tr>
</tbody>
</table>

The shift of N1 to N2 in $X_b, X$ is expressed as follows when one event arises.

$$X = \begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
1 \\
0
\end{pmatrix}, \quad X_b = \begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
1
\end{pmatrix}$$

Therefore the vectors $X, X_b$ in these cases are expressed as follows.

1. $$X = \begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
1
\end{pmatrix}, \quad X_b = \begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
1
\end{pmatrix}$$

2. $$X = \begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
1 \\
0
\end{pmatrix}, \quad X_b = \begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
1
\end{pmatrix}$$
\[ \begin{align*}
\text{(3) } X &= \begin{pmatrix}
0 \\
0 \\
0 \\
1 \\
0 \\
0
\end{pmatrix}, & X_b &= \begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
1
\end{pmatrix} \\
\text{(4) } X &= \begin{pmatrix}
0 \\
0 \\
1 \\
0 \\
0 \\
0
\end{pmatrix}, & X_b &= \begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
1
\end{pmatrix} \\
\text{(5) } X &= \begin{pmatrix}
0 \\
1 \\
0 \\
0 \\
0 \\
0
\end{pmatrix}, & X_b &= \begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
1
\end{pmatrix} \\
\text{(6) } X &= \begin{pmatrix}
1 \\
0 \\
0 \\
0 \\
0 \\
0
\end{pmatrix}, & X_b &= \begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
1
\end{pmatrix} \\
\text{(7) } X &= \begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
1
\end{pmatrix}, & X_b &= \begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
1
\end{pmatrix}
\end{align*} \]
$X = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad X_b = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$

$X = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad X_b = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$X = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad X_b = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$

$X = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad X_b = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$

$X = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad X_b = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$
$X = \begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
1
\end{pmatrix} \quad \quad \quad X_b = \begin{pmatrix}
0 \\
0 \\
0 \\
1 \\
0 \\
0
\end{pmatrix}$

$X = \begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
1 \\
0 \\
0
\end{pmatrix} \quad \quad \quad X_b = \begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
1 \\
0 \\
0
\end{pmatrix}$

$X = \begin{pmatrix}
0 \\
0 \\
0 \\
1 \\
0 \\
0 \\
0
\end{pmatrix} \quad \quad \quad X_b = \begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
1 \\
0 \\
0
\end{pmatrix}$

$X = \begin{pmatrix}
0 \\
0 \\
1 \\
0 \\
0 \\
0 \\
0
\end{pmatrix} \quad \quad \quad X_b = \begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
1 \\
0 \\
0
\end{pmatrix}$

$X = \begin{pmatrix}
0 \\
1 \\
0 \\
0 \\
0 \\
0 \\
0
\end{pmatrix} \quad \quad \quad X_b = \begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
1 \\
0
\end{pmatrix}$
$\begin{align*}
  X &= \begin{pmatrix}
    1 \\
    0 \\
    0 \\
    0 \\
    0 \\
  \end{pmatrix} \\
  X_b &= \begin{pmatrix}
    0 \\
    0 \\
    0 \\
    1 \\
    0 \\
  \end{pmatrix} \\
  X &= \begin{pmatrix}
    0 \\
    0 \\
    0 \\
    0 \\
    1 \\
  \end{pmatrix} \\
  X_b &= \begin{pmatrix}
    0 \\
    0 \\
    0 \\
    1 \\
    0 \\
  \end{pmatrix} \\
  X &= \begin{pmatrix}
    0 \\
    0 \\
    0 \\
    1 \\
    0 \\
  \end{pmatrix} \\
  X_b &= \begin{pmatrix}
    0 \\
    0 \\
    0 \\
    1 \\
    0 \\
  \end{pmatrix} \\
  X &= \begin{pmatrix}
    0 \\
    0 \\
    1 \\
    0 \\
    0 \\
  \end{pmatrix} \\
  X_b &= \begin{pmatrix}
    0 \\
    0 \\
    1 \\
    0 \\
    0 \\
  \end{pmatrix}
\end{align*}$
$X = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$X_b = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$X = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$X_b = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$

$X = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$

$X_b = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$

$X = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$

$X_b = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$
$\mathbf{X} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{X}_b = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$\mathbf{X} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{X}_b = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$\mathbf{X} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{X}_b = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$\mathbf{X} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$, $\mathbf{X}_b = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$\mathbf{X} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$, $\mathbf{X}_b = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$
Substituting these to equation (7), we obtain the following equation.
\[
\hat{A} = \begin{pmatrix}
2 & 0 & 1 & 1 & 1 & 0 \\
0 & 12 & 4 & 0 & 3 & 0 \\
2 & 2 & 125 & 13 & 11 & 12 \\
1 & 5 & 21 & 41 & 687 & 100 \\
2 & 1 & 12 & 33 & 105 & 537 \\
\end{pmatrix}
\begin{pmatrix}
9 & 0 & 0 & 0 & 0 & 0 \\
0 & 21 & 0 & 0 & 0 & 0 \\
0 & 0 & 178 & 0 & 0 & 0 \\
0 & 0 & 0 & 429 & 0 & 0 \\
0 & 0 & 0 & 0 & 693 & 0 \\
\end{pmatrix}
\begin{pmatrix}
2 & 0 & 1 & 1 & 1 & 0 \\
0 & 4 & 9 & 0 & 3 & 0 \\
2 & 2 & 125 & 13 & 11 & 12 \\
9 & 21 & 178 & 429 & 851 & 231 \\
9 & 21 & 178 & 429 & 851 & 63 \\
9 & 21 & 178 & 429 & 851 & 231 \\
9 & 21 & 178 & 429 & 851 & 231 \\
9 & 21 & 178 & 429 & 851 & 231 \\
9 & 21 & 178 & 429 & 851 & 231 \\
\end{pmatrix}
\begin{pmatrix}
9 & 0 & 0 & 0 & 0 & 0 \\
0 & 4 & 9 & 0 & 3 & 0 \\
2 & 2 & 125 & 13 & 11 & 12 \\
9 & 21 & 178 & 429 & 851 & 231 \\
9 & 21 & 178 & 429 & 851 & 63 \\
9 & 21 & 178 & 429 & 851 & 231 \\
9 & 21 & 178 & 429 & 851 & 231 \\
9 & 21 & 178 & 429 & 851 & 231 \\
9 & 21 & 178 & 429 & 851 & 231 \\
\end{pmatrix}
\]

Total number of upper shift from N1: 156
Total number of lower shift from N1: -
Total number of upper shift from N2: 59
Total number of lower shift from N2: 105
Total number of upper shift from N3: 14
Total number of lower shift from N3: 74
Total number of upper shift from N4: 5
Total number of lower shift from N4: 48
Total number of upper shift from N5: 0
Total number of lower shift from N5: 9
Total number of upper shift from N6: -
Total number of lower shift from N6: 7

We can observe that upper shift from N1 is dominant and on the contrary, lower shift form N3, N4, N5 and N6 is dominant. This implies that customers buy rather cheap goods at first for the trial, and after confirming the quality, they make upper shift in selecting brands. After reaching higher brands, they buy cheaper goods and that leads to a lower shift in brand selection.

Hearing from customers, we can also find that they buy necklace for themselves and confirm the quality. After that, they make gift by selecting upper brand. Sometimes they buy lower brand goods for themselves after making gift.

That scene can often be seen and the result shows its sequence well.
When the shop owner introduces new brand goods, he/she has to determine the price. If it is not reasonable, customers do not select their brand position as the shop owner assumes. These are confirmed by the brand shift transition, which forces the shop owner to re-consider the price and brand position of the new brand goods.

5. APPLICATION OF THIS METHOD

There may arise following case. Consumers and producers do not recognize brand position clearly. But analysis of consumers’ behavior let them know their brand position in the market. In such a case, strategic marketing guidance to select brand would be introduced.

Setting in order the brand position of various goods and taking suitable marketing policy, enhancement of sales would be enabled.

6. REMARKS

We have made an analysis before (Takeyasu et al., 2011).

Hereinafter, comparison with the previous research is executed, mainly focusing on Necklace/Pendant.

<table>
<thead>
<tr>
<th>Data gathering Period</th>
<th>Previous Research</th>
<th>Research in this paper</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>October 2006–May 2009</td>
<td>April 2008–May 2011</td>
</tr>
<tr>
<td></td>
<td>(32 months)</td>
<td>(38 months)</td>
</tr>
<tr>
<td>Order number</td>
<td>2438</td>
<td>4411</td>
</tr>
<tr>
<td>Total number of shifts in one genre</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Necklace/pendant</td>
<td>615</td>
<td>2181</td>
</tr>
<tr>
<td>Pierced earrings</td>
<td>117</td>
<td>567</td>
</tr>
<tr>
<td>Ring</td>
<td>161</td>
<td>1205</td>
</tr>
<tr>
<td>Bracelet/Bangle</td>
<td>22</td>
<td>458</td>
</tr>
</tbody>
</table>

We can observe that repeated purchasing has increased for several times compared with the former research period.

In particular, Bracelet/Bangle has increased tremendously compared with other genres. It is because shop owner has introduced new products of leathered pair Bracelet into the market and they made a great hit.

Now, we compare the contents of upper shift and lower shift in the field of Necklace/
Pendant.

<Previous Research>

<table>
<thead>
<tr>
<th></th>
<th>N6</th>
<th>N5</th>
<th>N4</th>
<th>N3</th>
<th>N2</th>
<th>N1</th>
<th>summary</th>
<th>Share(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper shift</td>
<td>–</td>
<td>1</td>
<td>7</td>
<td>20</td>
<td>49</td>
<td>71</td>
<td>148</td>
<td>24.1</td>
</tr>
<tr>
<td>Same Rank</td>
<td>9</td>
<td>10</td>
<td>18</td>
<td>36</td>
<td>150</td>
<td>118</td>
<td>341</td>
<td>55.4</td>
</tr>
<tr>
<td>movement</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lower shift</td>
<td>8</td>
<td>21</td>
<td>31</td>
<td>33</td>
<td>33</td>
<td>–</td>
<td>126</td>
<td>20.5</td>
</tr>
<tr>
<td>Summary</td>
<td>17</td>
<td>32</td>
<td>56</td>
<td>89</td>
<td>232</td>
<td>189</td>
<td>615</td>
<td></td>
</tr>
<tr>
<td>Share(%)</td>
<td>2.8</td>
<td>5.2</td>
<td>9.1</td>
<td>14.5</td>
<td>37.7</td>
<td>30.7</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<Research in this paper>

<table>
<thead>
<tr>
<th></th>
<th>N6</th>
<th>N5</th>
<th>N4</th>
<th>N3</th>
<th>N2</th>
<th>N1</th>
<th>summary</th>
<th>Share(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper shift</td>
<td>–</td>
<td>0</td>
<td>5</td>
<td>14</td>
<td>59</td>
<td>156</td>
<td>234</td>
<td>10.7</td>
</tr>
<tr>
<td>Same Rank</td>
<td>2</td>
<td>12</td>
<td>125</td>
<td>341</td>
<td>687</td>
<td>537</td>
<td>1704</td>
<td>78.1</td>
</tr>
<tr>
<td>movement</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lower shift</td>
<td>7</td>
<td>9</td>
<td>48</td>
<td>74</td>
<td>105</td>
<td>–</td>
<td>243</td>
<td>11.1</td>
</tr>
<tr>
<td>Summary</td>
<td>9</td>
<td>21</td>
<td>178</td>
<td>429</td>
<td>851</td>
<td>693</td>
<td>2181</td>
<td></td>
</tr>
<tr>
<td>Share(%)</td>
<td>0.4</td>
<td>1.0</td>
<td>8.2</td>
<td>19.7</td>
<td>39.0</td>
<td>31.8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Compared with the former one, same rank movement extended the share greatly.
We can observe that the share of upper shift and those of lower shift is nearly the same for both of the previous research period and this research period. But the share of upper shift has slightly decreased compared with the former one in each period share.

We can also observe that the share of N6 and N5 has decreased rather greatly and N3 has increased similarly to rather big.
This implies that down shift tendency is prevailing owing to the impact of deflation caused by Lehman Shock (September 2008), and it has a big, long time influence.
The shop owner comments that new sudden disaster of earthquake happened around the east Japan area when business became to have recovered from the Lehman Shock. It is often the case that unexpected events arise whenever we are relieved at the recovering condition.
We must cope with those at any rate.
7. CONCLUSION

It is often observed that consumers select upper class brand when they buy next time. Suppose that former buying data and current buying data are gathered. Also suppose that upper brand is located upper in the variable array. Then the transition matrix becomes upper triangular matrix under the supposition that former buying variables are set input and current buying variables are set output. There might be also the case that customers selected lower brand to seek suitable price when they had chosen higher brand. Then it might compose items of lower triangular matrix. Utilizing jewelry/accessory purchasing history recorded of on-line shopping over three years, above structure was investigated and confirmed. Comparison with our previous research was also executed. Some interesting results were obtained.

Unless planner for products does not notice its brand position whether it is upper or lower than other products, matrix structure makes it possible to identify those by calculating consumers’ activities for brand selection. Thus, this proposed approach enables to make effective marketing plan and/or establishing new brand.

Various fields should be examined hereafter.

In the end, we appreciate Ms. Momoko Yoshida for her helpful support of work.

REFERENCES