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Efficiency Rental Price of Capital and Efficiency Wage of Labor*

Shigeru Watanabe**

ABSTRACT: The relationship between tax evasion and efficiency wage has been analyzed in Watanabe (March 1996), Chang and Lai (April 1996), Watanabe (Jun 1996) and Laszlo (2004). Further, in Watanabe (Sep. 2010) the efficiency rental price of capital as well as the efficiency wage of labor has been analyzed. Asymmetric information concerning the quality of rented capital, especially in the case of second-hand capital, will exist. Therefore, if the money is paid informally in addition to formal or market rental price of capital, there may be a chance such that the more efficient capital can be rented, though the informal payment will not be reported to tax authority as adequate cost when the tax is calculated. The tax evasion is another aspect of informal activity which may be used for financing the informal additional payment. However, in Watanabe (Sep. 2010) the relationship between the above two informal activities has not been examined simultaneously. The main purpose of this paper is to analyze the case where the two informal activities are explicitly related. The second purpose of this paper is to analyze the relationship between the elasticity of the efficiency of the labor with respect to the wage rate and that of the efficiency of the labor with respect to the efficient capital per worker. Main results of this paper are in the following. The condition similar to Sollow condition does not hold with respect to the informal additional rental price of capital. The elasticity of the efficiency of the rented capital with respect to the informal additional rental price is not equal to 1, but less than 1. In this paper following three cases; a case in the absence of tax, a case in the presence of tax but in the absence of tax evasion, and a case in the presence of tax evasion for financing the informal payment have been examined. Second result of this paper is that the sum of the elasticity of the efficiency of the labor with respect to the wage rate and that of the efficiency of the labor with respect to the efficient capital per worker is equal to 1. Hence, the elasticity of the efficiency of the labor with respect to the wage rate is less than 1, which is the different result from that indicated by the Sollow condition concerning the efficiency wages. Further,

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since the sum of the elasticity of the efficiency of the labor with respect to the wage rate and that of the efficiency of the labor with respect to the efficient capital per worker is equal to 1, the higher the elasticity of the efficiency of the labor with respect to the efficient capital per worker, the lower the elasticity of the efficiency of the labor with respect to the wage rate. Hence, in the sector where the former elasticity with respect to the capital is high, the latter elasticity with respect to the wage rate will be low. Then if the latter elasticity with respect to the wage rate is the decreasing function of the wage rate then the wage rate in that sector will be high. If in the high technology sector the former elasticity with respect to the capital is high then the wage rate of the laborer who are working in that sector will be high, if the latter elasticity with respect to the wage rate is the decreasing function of the wage rate.

Key Words: Efficiency Wage, Efficiency Rental Price, Tax Evasion, Sollw Condition.

1 Introduction

A purpose of this paper is to analyze the efficiency rental price of capital as well as the efficiency wage of labor. In Watanabe (Sep. 2010) the efficiency rental price of capital as well as the efficiency wage of labor has been analyzed. Asymmetric information concerning the quality of rented capital, especially in the case of second-hand capital, will exist. Therefore, if the money is paid informally in addition to formal or market rental price of capital, there may be a chance such that the more efficient capital can be rented, though the informal payment will not be reported to tax authority as adequate cost when the tax is calculated.

On the other hand, the tax evasion is another aspect of informal activity which may be used for financing the informal additional payment. However, in Watanabe (Sep. 2010) the relationship between the above two informal activities has not been examined simultaneously. The main purpose of this paper is to analyze the case where the two informal activities are explicitly related. The second purpose of this paper is to analyze the relationship between the elasticity of the efficiency of the labor with respect to the wage rate and that of the efficiency of the labor with respect to the efficient capital per worker.

Main results of this paper are in the following. The condition similar to Sollow condition does not hold with respect to informal additional rental price of capital. The elasticity of the efficiency of the rented capital with respect to the informal additional rental price is not equal to 1, but less than 1. In this paper three cases: (i) a case in the absence of tax, (ii) a case in the presence of
Second result of this paper is that the sum of the elasticity of the efficiency of the labor with respect to the wage rate and that of the efficiency of the labor with respect to the efficient capital per worker is equal to 1. Hence the elasticity of the efficiency of the labor with respect to the wage rate is less than 1, which is the different result from that indicated by the Sollow condition concerning the efficiency wages.

Further, since the sum of the elasticity of the efficiency of the labor with respect to the wage rate and that of the efficiency of the labor with respect to the efficient capital per worker is equal to 1, the higher the elasticity of the efficiency of the labor with respect to the efficient capital per worker, the lower the elasticity of the efficiency of the labor with respect to the wage rate. Hence, in the sector where the former elasticity with respect to the capital is high, the latter elasticity with respect to the wage rate will be low. Then if the latter elasticity with respect to the wage rate is the decreasing function of the wage rate then the wage rate in that sector will be high. If in the high technology sector the former elasticity with respect to the capital is high then the wage rate of the laborer who are working in that sector will be high, if the latter elasticity with respect to the wage rate is the decreasing function of the wage rate.

In the following sections, three cases; (i) a case in the absence of tax, (ii) a case in the presence of tax but in the absence of tax evasion, (iii) and a case in the presence of tax evasion for financing the informal payment will be examined. The concluding remarks will be given in the last section.

2 A Simple Model of Efficiency Rental Price of Capital and Efficiency Wage of Labor in the Absence of Tax

In the absence of tax, profit $\pi$ is denoted by the following (1).

$$\pi = PQ\left( e\left( w, \frac{\tilde{e}(x)K}{L} \right) L \right)$$

$$- wL - (x + \tilde{r})K.$$  \hspace{1cm} (1)

where $P$ is price, $Q$ is production function, $K$ is capital, $\tilde{r}$ is given formal or market rental price, $x$ is informal additional rental price.
$L$ is labor, $w$ is wage rate, 
$\tilde{e}(x)$ is efficiency of capital, 
\[ e \left( w, \frac{\tilde{e}(x)K}{L} \right) \]
is efficiency of labor, which depends on wage rate and efficiency capital per labor.

Differentiating the profit with respect to capital, additional informal payment concerning rental 
price of capital, labor and wage rate yields the following first order conditions.

\[
\frac{\partial \pi}{\partial K} = PQ' e' \tilde{e} - x - \bar{r} = 0, 
\]
where \[ Q' \equiv \frac{dQ}{d(\cdot) L}, e' \equiv \frac{\partial e}{\partial \left( \frac{\tilde{e}(x)K}{L} \right)}. \]

\[
\frac{\partial \pi}{\partial x} = PQ' e' K \tilde{e}' - K = 0, 
\]
where \[ \tilde{e}' \equiv \frac{d\tilde{e}}{dx}. \]

\[
\frac{\partial \pi}{\partial L} = PQ' (e' \tilde{e} KL^{-1} + e) - w = 0, 
\]
\[
\frac{\partial \pi}{\partial w} = PQ' e'. L - L = 0, 
\]
where \[ e' \equiv \frac{\partial e}{\partial w}. \]

Second order conditions are assumed to be satisfied.

From the first order conditions, the elasticity $\eta\tilde{e}$ of the efficiency of the rented capital with respect 
to the informal additional rental price is obtained.

\[
\eta\tilde{e} = \frac{x}{\tilde{e}} \tilde{e}' = 1 - \frac{\bar{r} \tilde{e}'}{\tilde{e}} < 1. 
\]

Hence, the elasticity of the efficiency of the rented capital with respect to the informal rental 
price is derived to be less than 1.
In the same way, the following relation can also be obtained.

\[ \eta^e_w + \eta^e_K = 1. \tag{7} \]

where

\[ \eta^e_w \equiv \frac{w}{e} \frac{\partial e}{\partial w}, \quad \eta^e_K \equiv \frac{w}{e} \frac{\partial e}{\partial (\frac{K}{L})}. \]

Hence, the sum of the elasticity of the efficiency of the labor with respect to the wage rate and that of the efficiency of the labor with respect to the efficient capital per worker is equal to 1.

Therefore, the following result can also be obtained; the elasticity of the efficiency of the labor with respect to the wage rate is less than 1, which is the different result from that derived by the Sollow condition concerning the efficiency wages.

### 3 A Simple Model of Efficiency Rental Price of Capital and Efficiency Wage of Labor in the Presence of Tax but in the Absence of Tax Evasion

In the presence of tax but in the absence of tax evasion, profit \( \pi \) is denoted by the following (8),

\[ \pi = PQ \left( e \left( w, \frac{\bar{e}(x)K}{L} \right) L \right) - wL - (x + \bar{r})K - t \left( PQ \left( e \left( w, \frac{\bar{e}(x)K}{L} \right) L \right) - wL - \bar{r}K \right), \tag{8} \]

where \( xK \) is not regarded as adequate cost for calculating tax.

Maximizing \( \pi \) with respect to \( K, x, L \) and \( w \) yields the following first order conditions.

\[ \frac{\partial \pi}{\partial K} = (1 - t) PQ' e^e \bar{e} - (x + \bar{r}) + t\bar{r} = 0, \tag{9} \]

\[ \frac{\partial \pi}{\partial x} = (1 - t) PQ' e^e \bar{e} \bar{K} - K = 0. \tag{10} \]

\[ \frac{\partial \pi}{\partial L} = (1 - t) PQ' (-e^e \bar{e} K L^{-1} + e) \]

\[ - w + tw \]

\[ = 0. \tag{11} \]
Second order conditions are assumed to be satisfied. From the first order conditions the following relations can straightforwardly be obtained.

\[ \eta^e_1 = 1 - (1 - t) \frac{\bar{e}}{\bar{e}} < 1, \]  
and

\[ \eta^e + \eta^e \frac{\bar{e}}{L} = 1. \]  

Hence, the following same results as those obtained in the absence of tax have been obtained, even if the informal additional payment can not be regarded as the adequate cost when the tax is calculated. The elasticity of the efficiency of the rented capital with respect to the informal rental price is derived to be less than 1. On the other hand, the sum of the elasticity of the efficiency of the labor with respect to the wage rate and that of the efficiency of the labor with respect to the efficient capital per worker is equal to 1, then the elasticity of the efficiency of the labor with respect to the wage rate is less than 1, which is the different result from that derived by the Sollow condition concerning the efficiency wages.

4 A Simple Model of Efficiency Rental Price of Capital and Efficiency Wage of Labor in the Presence of Tax Evasion

In Watanabe (Sep. 2010) the relationship between the two informal activities i.e. tax evasion and the informal additional payment for efficient capital, has not been examined simultaneously. In this section, it is explicitly assumed that the informal additional payment for efficient capital is financed by the tax evasion. Further, in Watanabe (Sep. 2010), in order to focus on the efficiency rental price of capital, the wage rate and the employment level of the labor were assumed to be given. In this section, the wage rate and the employment level are endogenous as well as the informal additional payment for efficient capital and the proceeds understatement rate.

4-1 A Case of Constant Probability of Detection

The probability \( q \) of tax evasion being detected is assumed to be given for simplicity. Then the expected profit is denoted by the following equation (15).
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\[ E\pi = \left\{ 1 - t \left( 1 - \frac{xK}{tPQ} \right) - \frac{qFt}{tPQ} \right\} PQ \]
\[ - wL - (x + \tilde{r}) K \]
\[ + t (wL + \tilde{r}K). \]  \hspace{1cm} (15)

where \( F \) is the penalty rate of tax evasion and the informal additional payment for efficient capital is financed by the tax evasion such that \( xK = t\epsilon PQ \), where \( \epsilon \) is the rate of proceeds understatement.

Maximizing (15) with respect to \( K, x, L, w \) yields the following first order conditions.

\[ \frac{\partial E\pi}{\partial K} = (1 - t) PQ' e\tilde{e} \]
\[ - \tilde{q}Fx \]
\[ - (1 - t) \tilde{r} \]
\[ = 0, \] \hspace{1cm} (16)

\[ \frac{\partial E\pi}{\partial x} = (1 - t) PQ' e\tilde{e} K\tilde{e}' \]
\[ - \tilde{q}FK \]
\[ = 0, \] \hspace{1cm} (17)

\[ \frac{\partial E\pi}{\partial L} = (1 - t) PQ' \{ e\tilde{e} K(-1) L^{-1} + e \} \]
\[ - (1 - t) w \]
\[ = 0, \] \hspace{1cm} (18)

\[ \frac{\partial E\pi}{\partial w} = (1 - t) PQ' e\tilde{e} L \]
\[ - (1 - t) L \]
\[ = 0, \] \hspace{1cm} (19)

Second order conditions are assumed to be satisfied.

From the first order conditions the following relations can straightforwardly be obtained.

\[ \eta\tilde{e} = 1 - \frac{\tilde{e}'(1 - t) \tilde{r}}{\tilde{e} F} < 1. \] \hspace{1cm} (20)
Therefore, also in this case the following results can be obtained: the elasticity of the efficiency of the rented capital with respect to the informal rental price is derived to be less than 1. and the sum of the elasticity of the efficiency of the labor with respect to the wage rate and that of the efficiency of the labor with respect to the efficient capital per worker is equal to 1, then the elasticity of the efficiency of the labor with respect to the wage rate is less than 1, which is the different result from that derived by the Solow condition concerning the efficiency wages.

4–2. A Case of Variable Probability of Detection

The probability of the tax evasion being detected is assumed to be an increasing function of ε. Then the expected profit is denoted by the following equation (22).

\[
E\pi = \left\{ 1 - t \left( 1 - \frac{xK}{tPQ} \right) - Ft \frac{x^2K^2}{t^2P^2Q^2} \right\} PQ \\
- wL - (x + \bar{r}) K \\
+ t (wL + \bar{r}K),
\]

where the informal additional payment for efficient capital is assumed to be financed by the tax evasion \(xK = t\varepsilon PQ\) and the function of the probability of the tax evasion being detected is specified such that \(q(\varepsilon) = \varepsilon\).

Maximizing (22) with respect to \(K, x, L, w\) yields the following first order conditions.

\[
\frac{\partial E\pi}{\partial K} = (1 - t) PQ' e\varepsilon \tilde{e} \\
- Fx^2 \frac{1}{P} \left( 2K^2 - K^2Q^2Q' e\varepsilon \tilde{e} \right) \\
- (1 - t) \bar{r} \\
= 0,
\]

(23)
Second order conditions are assumed to be satisfied. From the first order conditions the following relations can straightforwardly be obtained.

\[
\frac{\partial E}{\partial x} = (1-t)PQ\cdot e_2 K\bar{e} - F\frac{1}{t}K^2 \frac{1}{P} \left\{ 2x \frac{1}{Q} - x^2 Q^2 Q' e_2 K\bar{e} \right\} = 0, \tag{24}
\]

\[
\frac{\partial E}{\partial L} = (1-t)PQ' \left\{ e_2 \bar{e} K(-1)L^{-1} + e \right\} + \frac{F}{t}x^2 K^2 \frac{1}{P} Q^2 Q' e_2 K(-1)L^{-1} + e \right\} - (1-t)w = 0, \tag{25}
\]

\[
\frac{\partial E}{\partial w} = (1-t)PQ' e_1 L + \frac{F}{t}x^2 K^2 \frac{1}{P} Q^2 Q' e_1 L - (1-t)L = 0. \tag{26}
\]

Therefore, the elasticity of the efficiency of the rented capital with respect to the informal rental price is obtained to be less than 1. and the sum of the elasticity of the efficiency of the labor with respect to the wage rate and that of the efficiency of the labor with respect to the efficient capital per worker is equal to 1, then the elasticity of the efficiency of the labor with respect to the wage rate is less than 1, which is the different result from that derived by the Sollow condition concerning the efficiency wages.
5 Concluding Remarks

The relationship between tax evasion and efficiency wage has been analyzed in Watanabe (March 1996), Chang and Lai (April 1996), Watanabe (Jun 1996) and Laszlo (2004). Further, in Watanabe (Sep. 2010) the efficiency rental price of capital as well as the efficiency wage of labor has been analyzed. Asymmetric information concerning the quality of rented capital, especially in the case of second-hand capital, will exist. Therefore, if the money is paid informally in addition to formal or market rental price of capital, there may be a chance such that the more efficient capital can be rented, though the informal payment can not be reported to tax authority as adequate cost when the tax is calculated.

On the other hand, the tax evasion is another aspect of informal activity which may be used for financing the informal additional payment. However, in Watanabe (Sep. 2010) the relationship between the above two informal activities has not been examined simultaneously. The main purpose of this paper is to analyze the case where the two informal activities are explicitly related. The second purpose of this paper is to analyze the relationship between the elasticity of the efficiency of the labor with respect to the wage rate and that of the efficiency of the labor with respect to the efficient capital per worker.

Main results of this paper are in the following. The condition similar to Sollow condition does not hold with respect to informal additional rental price of capital. The elasticity of the efficiency of the rented capital with respect to the informal additional rental price is not equal to 1, but less than 1. In this paper the following three cases; (i) a case in the absence of tax, (ii) a case in the presence of tax but in the absence of tax evasion, (iii) and a case in the presence of tax evasion for financing the informal payment have been examined.

Second result of this paper is that the sum of the elasticity of the efficiency of the labor with respect to the wage rate and that of the efficiency of the labor with respect to the efficient capital per worker is equal to 1. Hence, the elasticity of the efficiency of the labor with respect to the wage rate is less than 1, which is the different result from that derived by the Sollow condition concerning the efficiency wages.

Since the sum of the elasticity of the efficiency of the labor with respect to the wage rate and that of the efficiency of the labor with respect to the efficient capital per worker is equal to 1, the higher the elasticity of the efficiency of the labor with respect to the efficient capital per worker, the lower the elasticity of the efficiency of the labor with respect to the wage rate. Hence, in the sector where the former elasticity with respect to the capital is high, the latter elasticity with
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respect to the wage rate will be low. Then if the latter elasticity with respect to the wage rate is the decreasing function of the wage rate then the wage rate in that sector will be high. If in the high technology sector the former elasticity with respect to the capital is high then the wage rate of the laborer who are working in that sector will be high, if the latter elasticity with respect to the wage rate is the decreasing function of the wage rate.

Notes


References