<table>
<thead>
<tr>
<th>Title</th>
<th>A Note on Efficiency Wage and O.J.T.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Author(s)</td>
<td>Watanabe, Shigeru; Kubota, Kazuo; Koh, Genju</td>
</tr>
<tr>
<td>Editor(s)</td>
<td></td>
</tr>
<tr>
<td>Citation</td>
<td>大阪府立大学経済研究 2010年3月号 p.117-123</td>
</tr>
<tr>
<td>Issue Date</td>
<td>2010-12-24</td>
</tr>
<tr>
<td>URL</td>
<td><a href="http://repository.osakafu-u.ac.jp/dspace/">http://repository.osakafu-u.ac.jp/dspace/</a></td>
</tr>
<tr>
<td>Rights</td>
<td></td>
</tr>
</tbody>
</table>
A Note on Efficiency Wage and O.J.T.

Shigeru Watanabe*
Kazuo Kubota**
Genju Koh***

ABSTRACT: A purpose of this note is to analyze the relationship between efficiency wage and O.J.T. The following results have been derived. The elasticity of efficiency with respect to wage rate concerning the workers who are not given the opportunity of the O.J.T. is equal to 1, however, that elasticity concerning the workers who are given that opportunity is less than 1. In this note, how to select the workers who are given that opportunity of the O.J.T. is not examined but it is assumed that the selection process is adequately done from the point of equity among workers.

The ratio between the efficiency of the workers with O.J.T. and that of the workers without O.J.T. has been derived to be higher than the ratio between the wage rate of the former workers and that of the latter workers.

If the wage rate is assumed to be same among workers regardless of O.J.T., though the efficiency depends on the O.J.T., the following results can also be obtained.

If the effect of wage rate on the efficiency of the workers with O.J.T. is larger than that effect on the workers without O.J.T. then the elasticity of the efficiency of workers without O.J.T. with respect to the wage rate is less than 1. If the former effect is less than the latter effect, the elasticity is higher than 1. If the former effect is equal to the latter effect, then the elasticity becomes equal to 1.

On the other hand, the elasticity of the efficiency of workers who are given the opportunity of O.J.T. with respect to the wage rate can not be determined in general, though if the elasticity of the workers without O.J.T. is 1, then the elasticity of the efficiency of workers with O.J.T. with respect to the wage rate is less than 1.

* Professor at the University of Osaka Prefecture, College of Economics, 1-1, Gakuencho, Nakaku, Sakai-City, Osaka 599-8531, Japan
** Postgraduate Student at the University of Osaka Prefecture, College of Economics
***Physician at the Osaka Saiseikai Noe Hospital
The elasticity of the efficiency of workers with O.J.T. with respect to the cost of O.J.T. has been derived to be less than 1. \textit{[THE JOURNAL OF ECONOMIC STUDIES, Published in December 2010]}

\textbf{Key Words:} Efficiency Wage, O.J.T.

1 Introduction

A purpose of this note is to analyze the relationship between efficiency wage\(^1\) and O.J.T.\(^2\). The following results have been derived. The elasticity of efficiency with respect to wage rate concerning the workers who are not given the opportunity of the O.J.T. is equal to 1, however, that elasticity concerning the workers who are given that opportunity is less than 1. In this note, how to select the workers who are given that opportunity of the O.J.T. is not examined but it is assumed that the selection process is adequately done from the point of equity among workers.

The ratio between the efficiency of the workers with O.J.T. and that of the workers without O.J.T. has been derived to be higher than the ratio between the wage rate of the former workers and that of the latter workers.

If the wage rate is assumed to be same among workers regardless of O.J.T., though the efficiency depends on the O.J.T., the following results have also been obtained.

If the effect of wage rate on the efficiency of the workers with O.J.T. is larger than that effect on the workers without O.J.T. then the elasticity of the efficiency of workers without O.J.T. with respect to the wage rate is less than 1. If the former effect is less than the latter effect, the elasticity is higher than 1. If the former effect is equal to the latter effect, then the elasticity becomes equal to 1.

On the other hand, the elasticity of the efficiency of workers who are given the opportunity of O.J.T. with respect to the wage rate can not be determined in general, though if the elasticity of the workers without O.J.T. is 1, then the elasticity of the efficiency of workers with O.J.T. with respect to the wage rate is less than 1.

The elasticity of the efficiency of workers with O.J.T. with respect to the cost of O.J.T. has been derived to be less than 1.

In the next section a simple model will be shown. Concluding remarks will be given in the last section.
2 A Simple Model

In the first case, the wage rate is assumed to be same regardless of O.J.T. In this case profit is denoted by the following (1).

\[ \pi = p \alpha \log \left\{ e(w) \left( l - m \right) + \hat{e}(w, r) m \right\} - wl - rm, \]  

(1)

where \( p \) is the price, \( \alpha \log \left\{ e(w) \left( l - m \right) + \hat{e}(w, r) m \right\} \) is the production function, \( e \) is the efficiency of workers who are not given the opportunity of the O.J.T., \( l \) is the total number of the employed workers, \( m \) is the number of the employed workers who are given the opportunity of the O.J.T., \( \hat{e} \) is the efficiency of the workers who are given the opportunity of the O.J.T. and \( r \) is the cost of the O.J.T. per worker.

Maximizing (1) with respect to \( l, w, m \) and \( r \) yields the following first order conditions.

\[ \frac{\partial \pi}{\partial l} = p \alpha \frac{e(w)}{e(w) \left( l - m \right) + \hat{e}(w, r) m} - w = 0, \]  

(2)

\[ \frac{\partial \pi}{\partial w} = p \alpha \frac{\frac{de}{dw} \left( l - m \right) + \frac{\partial \hat{e}}{\partial w} m}{e(w) \left( l - m \right) + \hat{e}(w, r) m} - l = 0, \]  

(3)

\[ \frac{\partial \pi}{\partial m} = p \alpha \frac{-e(w) + \hat{e}(w, l)}{e(w) \left( l - m \right) + \hat{e}(w, r) m} - r = 0, \]  

(4)

\[ \frac{\partial \pi}{\partial r} = p \alpha \frac{\frac{\partial \hat{e}}{\partial r} \frac{m}{m}}{e(w) \left( l - m \right) + \hat{e}(w, r) m} - m = 0. \]  

(5)

Second order conditions are assumed to be satisfied.

From the first order conditions the following relations can straightforwardly be obtained.

\[ \eta_w^e \equiv \frac{de}{dw} \frac{e}{w} = \frac{1}{1 + \frac{m}{l} \left( \frac{\frac{\partial \hat{e}}{\partial w}}{\frac{de}{dw}} - 1 \right)}. \]  

(6)
Hence, from (6) the elasticity of the efficiency of workers who are not given the opportunity of the O.J.T. with respect to the wage rate will depend on the following condition.

$\eta_w^e = \frac{\partial \hat{e}}{\partial w} = \frac{e}{\hat{e}} \left\{ \frac{l}{m} (1 - \eta_w^e) + \eta_w^e \right\}.$

(7)

$\eta_r^e = \frac{\partial \hat{e}}{\partial r} = 1 - \frac{e}{\hat{e}} < 1.$

(8)

Hence, if the effect of increased wage rate on the efficiency of the workers who are given the opportunity of O.J.T. is larger than that effect on the workers who are not given the opportunity of the O.J.T. then the elasticity of the efficiency of workers who are not given the opportunity of the O.J.T. with respect to the wage rate is less than 1. If the former effect is less than the latter effect, the elasticity is larger than 1. And if the former effect is equal to the latter effect, then the elasticity becomes equal to 1.

On the other hand, the elasticity of the efficiency of workers who are given the opportunity of O.J.T. with respect to the wage rate can not be determined in general, though if the elasticity of the workers who are not given the opportunity of the O.J.T. is 1, then the elasticity of the efficiency of workers who are given the opportunity of O.J.T. with respect to the wage rate is less than 1.

The elasticity of the efficiency of workers who are given the opportunity of the O.J.T. with respect to the cost of O.J.T. instead of the wage rate is derived to be less than 1.

More realistic case where the wage rate of the workers with O.J.T. is not equal to that of the workers without O.J.T. will be examined in the following.

In this case the profit is denoted by the following equation.
where $\hat{w}$ is the wage rate of the workers who are given the opportunity of the O.J.T. and $w$ is the wage rate of the workers who are not given that.

Maximizing the equation with respect to $l$, $w$, $m$, $\hat{w}$ and $r$ yields the following first order conditions.

\[
\frac{\partial \pi}{\partial l} = p\alpha \frac{e}{e(l-m) + \hat{e}m} - w = 0, \quad (11)
\]

\[
\frac{\partial \pi}{\partial w} = p\alpha \frac{e'(l-m)}{e(l-m) + \hat{e}m} - (l-m) = 0, \quad (12)
\]

\[
\frac{\partial \pi}{\partial m} = p\alpha \frac{-e + \hat{e}}{e(l-m) + \hat{e}m} + w - \hat{w} - r = 0, \quad (13)
\]

\[
\frac{\partial \pi}{\partial \hat{w}} = p\alpha \frac{\hat{e}'m}{e(l-m) + \hat{e}m} = 0, \quad (14)
\]

\[
\frac{\partial \pi}{\partial r} = p\alpha \frac{\hat{e}'m}{e(l-m) + \hat{e}m} - m = 0. \quad (15)
\]

In the same way, the following relations can straightforwardly be obtained from the first order conditions.

\[
\eta_{\hat{w}}^e = \frac{de}{d\hat{w}} w = 1. \quad (16)
\]

\[
\eta_{w}^e = \frac{d\hat{e}}{d\hat{w}} w = \frac{e}{\hat{e}} \frac{\hat{e}'m}{w - 1 + \hat{e}} = \frac{\hat{e}(\hat{e}-1)}{w \hat{w}' - 1 + \hat{e}} = -\frac{\hat{e}}{\hat{e}'} < 1. \quad (17)
\]

where $\frac{\hat{e}}{\hat{e}} = \frac{\hat{w}}{\hat{w}} + \frac{r}{\hat{w}}$ from the first order conditions, hence $\frac{\hat{e}}{\hat{e}} > \frac{\hat{w}}{\hat{w}}$, then $\frac{\hat{e}}{\hat{e}} < \frac{\hat{w}}{\hat{w}}$.

Therefore, the ratio, $\frac{\hat{e}}{\hat{e}}$, between the efficiency of the workers with O.J.T. and that of the workers without O.J.T. becomes higher than the ratio, $\frac{\hat{w}}{\hat{w}}$, between the wage rate of the former workers and that of the latter workers.
\[ \eta_r \equiv \frac{\partial \hat{e}}{e \partial r} = \frac{1 - \frac{e}{\hat{e}}}{\hat{w} - \frac{w}{r} + 1} = \frac{\frac{e}{\hat{e}} \left( \frac{\hat{e}}{e} - 1 \right)}{\frac{w}{r} \left( \frac{\hat{w}}{w} - 1 + \frac{r}{w} \right)} = \frac{\frac{e}{\hat{e}}}{\frac{w}{r}} < 1, \]  

where \( \frac{\hat{e}}{e} = \hat{w} + \frac{r}{w} \) from the first order conditions, hence \( \frac{\hat{e}}{e} > \frac{r}{w} \), then \( \frac{e}{\hat{e}} < \frac{w}{r} \).

Second order conditions are assumed to be satisfied.

Hence, the elasticity of the efficiency of workers who are not given the opportunity of the O.J.T. with respect to the wage rate is equal to 1 and the elasticity of the efficiency of workers who are given the opportunity of the O.J.T. with respect to the wage rate is less than 1. And the elasticity of the efficiency of workers who are given the opportunity of the O.J.T. with respect to the cost of O.J.T. instead of the wage rate is derived to be less than 1.

3 Concluding Remarks

A purpose of this note is to analyze the relationship between efficiency wage and O.J.T.. The following results have been derived. The elasticity of efficiency with respect to wage rate concerning the workers who are not given the opportunity of the O.J.T. is equal to 1, however, that elasticity concerning the workers who are given that opportunity is less than 1. In this note, how to select the workers who are given that opportunity of the O.J.T. is not examined but it is assumed that the selection process is adequately done from the point of equity among workers.

The ratio between the efficiency of the workers with O.J.T. and that of the workers without O.J.T. has been derived to be higher than the ratio between the wage rate of the former workers and that of the latter workers.

If the wage rate is assumed to be same among workers regardless of O.J.T., though the efficiency depends on the O.J.T., the following results have also been obtained.

If the effect of wage rate on the efficiency of the workers with O.J.T. is larger than that effect on the workers without O.J.T. then the elasticity of the efficiency of workers without O.J.T. with respect to the wage rate is less than 1. If the former effect is less than the latter effect, the elasticity is higher than 1. If the former effect is equal to the latter effect, then the elasticity becomes equal to 1.
On the other hand, the elasticity of the efficiency of workers who are given the opportunity of O.J.T. with respect to the wage rate can not be determined in general, though if the elasticity of the workers without O.J.T. is 1, then the elasticity of the efficiency of workers with O.J.T. with respect to the wage rate is less than 1.

The elasticity of the efficiency of workers with O.J.T. with respect to the cost of O.J.T. has been derived to be less than 1.

2 See Mincer (1962)

References